

PROPERTIES OF THE REGULAR SPECTRUM OF ELEMENTS IN TOPOLOGICAL ALGEBRAS

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Abstract. Relations between the usual spectrum of elements and the regular (or extended) spectrum of elements of topological algebras are described. Conditions when the spectral radius, the regular spectral radius and the radius of boundedness of elements in topological (not necessarily unital and locally convex) algebras coincide are given.

Let A be a *topological algebra* over \mathbb{C} with separately continuous multiplication (in short, a topological algebra). In particular, when the multiplication, as a map $A \times A \rightarrow A$, is continuous, we speak about a *topological algebra with jointly continuous multiplication*.

Let $\text{Inv}A$ denote the set of all invertible elements in A and $\text{Qinv}A$ the set of all quasi-invertible elements in A (that is, of elements $a \in A$, for which there is an element a_q^{-1} (the *quasi-inverse* of a) such that $a + a_q^{-1} = aa_q^{-1} = a_q^{-1}a$). A topological algebra A is called a *Q-algebra*, if the set $\text{Qinv}A$ (for unital algebras $\text{Inv}A$) is open in A . Moreover, a topological algebra A is *locally complete* (in the locally convex case, G. R. Allan used in [4], p. 401, the term *pseudo-complete*), if every subalgebra of A , generated by a closed, bounded, idempotent and absolutely pseudoconvex subset U , is complete in the normed topology (let us remind that U is *idempotent*, if $UU \subset U$, *absolutely k-convex*, if

$$U = \Gamma_k(U) = \left\{ \sum_{v=1}^n \alpha_v u_v : n \in \mathbb{N}, u_1, \dots, u_n \in U, \alpha_1, \dots, \alpha_n \in \mathbb{C}, \sum_{v=1}^n |\alpha_v|^k \leq 1 \right\}$$

and *absolutely pseudoconvex*, if U is absolutely k -convex for some $k \in (0, 1]$). In addition, A is a *locally convex* (*locally k-convex* or *locally m-convex*) *algebra* if A has a base of neighbourhoods of zero, consisting of absolutely convex (respectively, absolutely

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