NEAR EXACT OPERATOR BANACH FRAMES IN BANACH SPACES

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Abstract. Near exact operator Banach frame (near exact OBF) is defined and studied. Examples have been given for the existence of near exact operator Banach frames. Also, a sufficient condition for an operator Banach frame to be a near exact OBF is given. Further, it has been proved that if $E$ and $F$ are Banach spaces having near exact operator Banach frames, then the product space $E \times F$ also has a near exact operator Banach frame. Further, we consider block perturbation of operator Banach frames and prove that a block perturbation of an OBF is also an OBF. Finally, we give an application of near exact OBF related to eigenvalue problem.

1. Introduction

Frames are the redundant systems introduced by Duffin and Schaeffer [9], to study some deep problems in non-harmonic Fourier series. For a nice introduction to frames, one may refer to [7]. Frames were extended to Banach spaces by Feichtinger and Gröchenig [11], who, in fact, introduced the notion of atomic decompositions for Banach spaces. Later, Gröchenig [12] introduced a more general concept called Banach frame for Banach spaces. He gave the following definition of a Banach frame.

Let $E$ be a Banach space and $E_d$ be an associated Banach space of scalar valued sequences indexed by $\mathbb{N}$. Let $\{f_n\} \subset E^*$ and $S: E_d \to E$ be given. The pair $(\{f_n\}, S)$ is called a Banach frame for $E$ with respect to $E_d$, if

1. $\{f_n(x)\} \in E_d, \ x \in E$;
2. there exist constants $A$ and $B$ with $0 < A \leq B < \infty$ such that

$$A\|x\|_E \leq \|\{f_n(x)\}\|_{E_d} \leq B\|x\|_E, \ x \in E;$$

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