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A NOTE ON WEINSTEIN TRANSFORM ON PRODUCTS OF CENTRAL MORREY SPACES

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Abstract. We prove boundedness of convolution operators given by product domain versions of the Weinstein transform acting on products of central weighted Morrey spaces.

1. Introduction

The aim of this note is to obtain boundedness results on some function spaces of convolution operators induced by a product version of the kernel $\mathscr{K}_\alpha$, defined by

$$\mathscr{K}_\alpha\left(y,s\right) := \frac{\Gamma\left(\left(\alpha+n+1\right)/2\right)}{\Gamma\left(\left(\alpha+1\right)/2\right)\pi^{n/2}} \frac{s^{\alpha+1}}{\left(\left|y\right|^2+s^2\right)^{(\alpha+n+1)/2}} \tag{1.1}$$

where $\alpha > -1$ and $(y,s) \in \mathbb{R}^{n+1}_+$.

This kernel is related to the elliptic partial differential equation

$$D_\alpha u := s^{-\alpha}\left(\frac{\partial^2 u}{\partial y_1^2} + ... + \frac{\partial^2 u}{\partial y_n^2} + \frac{\partial^2 u}{\partial s^2} - \frac{\alpha}{s}\frac{\partial u}{\partial s}\right) = 0 \tag{1.2}$$

with $\alpha > -1$. Solutions to (1.2) are called generalized axially symmetric potentials (cf. [13]). Notice that, for $\alpha = 0$ we recover the Laplace equation.

Several authors have approached the study of boundary value problems related to this kernel, as well as modified versions of it (see, for example, [14],[2],[3], [6]).

As it happens in the case of the Poisson kernel for the upper half-space, it can also be proved that: (i) $\left\|\mathscr{K}_{\alpha,s}\right\|_{L^1} = 1$, where $\mathscr{K}_{\alpha,s}\left(y\right) := \mathscr{K}_\alpha\left(y,s\right)$, (ii) $\mathscr{K}_{\alpha,s} \to \delta_0$ in $\mathscr{S}'$ as $s \to 0^+$, and (iii) $\mathscr{K}_\alpha$ is a solution to the equation (1.2) in $\mathbb{R}^{n+1}_+$ (see [14]). In that article, generalizing techniques developed in [3] and [2], the author characterizes a

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