

THE NATURAL ORDERING IN STRICTLY REAL m -CONVEX Q -ALGEBRAS

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Abstract. J. B. Miller considered orders in strictly real Banach algebras. The aim of this paper is to extend the basic results about the natural ordering to strictly real Hausdorff m -convex ones. In this frame, one is led to put the additional condition that they are Q -algebras, which is, anyway, satisfied in the Banach case. The results about the associated order are also valid in this framework but they are not included here.

1. Introduction

In 1949, I. Kaplansky has introduced in [12] the class of strictly real Banach algebras. Since then, not much work has been done on them. In [16], J. B. Miller considers what he calls the natural ordering in a strictly real Banach algebra. The new idea is to use the principal component G_1 . He also introduced what he called the associated order. Here we work with unital strictly real m -convex Hausdorff Q -algebras. From section 5 onwards we intend, using the results of sections 3 and 4, to extend to this class of algebras the results of Miller concerning the natural order. Those relative to the associated order can also be extended to our framework, but we do not include them here because their proofs are the same, although some of these can be simplified using our results of Section 3.

In Theorem 3.9 we show that if E is a unital strictly real m -convex Hausdorff Q -algebra, then the spectrum of any $x \in E$ is the image of its Gelfand transform, as it happens in the case of unital m -convex commutative complex Hausdorff Q -algebras. In the aforementioned theorem and Corollary 4.4 we also prove that the same is true for other algebras related with E , such as its complexification $E_{\mathbb{C}}$ and E modulo its

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