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Duals of a frame in quaternionic Hilbert spaces

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Abstract. Frames in a separable quaternionic Hilbert space were introduced and studied in [17] to have more applications. In this paper, we extend the study of frames in quaternionic Hilbert spaces and introduce different types of duals of a frame in separable quaternionic Hilbert spaces. As an application, we give the orthogonal projection of $\ell^2(\mathfrak{Q})$ onto the range of analysis operator of the given frame, in terms of elements of canonical dual frame and elements of the frame in quaternionic Hilbert space. Finally, we give an expression for the orthogonal projection in terms of operators related to the frame and its canonical dual frame in quaternionic Hilbert space.

1. Introduction

Formally, frames for Hilbert spaces (in particular for $L^2[a,b]$) were introduced way back in 1952 by Duffin and Schaeffer [11] as a tool to study of non-harmonic Fourier series. They defined the following

"A sequence $\{x_n\}_{n\in\mathbb{N}}$ in a Hilbert space $\mathcal{H}$ is said to be a *frame* for $\mathcal{H}$ if there exist constants A and B with $0 < A \leq B < \infty$ such that

$$A\|x\|^2 \leq \sum_{n=1}^{\infty} |\langle x, x_n\rangle|^2 \leq B\|x\|^2, \quad \text{for all } x \in \mathcal{H}.\text{"} \tag{1.1}$$

Moreover, the positive constants A and B, respectively, are called *lower* and *upper* frame bounds for the frame $\{x_n\}_{n\in\mathbb{N}}$. The inequality (1.1) is called the *frame inequality* for the frame $\{x_n\}_{n\in\mathbb{N}}$. A sequence $\{x_n\}_{n\in\mathbb{N}} \subset \mathcal{H}$ is called a *Bessel sequence* if it satisfies upper frame inequality in (1.1). A frame $\{x_n\}_{n\in\mathbb{N}}$ in $\mathcal{H}$ is said to be

- *tight* if it is possible to choose $A,\ B$ satisfying inequality (1.1) with $A = B$.

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