

Notes on lifting of Bertrand curve on tangent space TR^3

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Abstract. In this paper, firstly, we define the Bertrand curve of any curve with respect to the vertical, complete and horizontal lifts on space R^3 to its tangent space $TR^3 = R^6$. Secondly, we examine the Frenet-Serret apparatus $\{T^*(s), N^*(s), B^*(s), \kappa^*, \tau^*\}$ and the Darboux vector W^* of the Bertrand curve α^* according to the vertical, complete and horizontal lifts on TR^3 by depend on the lifting of Frenet-Serret apparatus $\{T(s), N(s), B(s), \kappa, \tau\}$ of the first curve α on space R^3 . As a result of this transformation on space R^3 to its tangent space TR^3 , we can speak about the features of Bertrand curve of any curve on space TR^3 by looking at the characteristics of the first curve.

1. Introduction

In differentiable geometry, lift method has an important role. Because, it is possible to generalize to differentiable structures on any space (resp. manifold) to extended spaces (resp. extended manifolds) using lift function [4, 5, 6, 7, 8, 10]. Thus, it may be extended the following theorem given on space R^3 to its tangent space TR^3 . It is well known that many studies related to the differential geometry of curves have been made. Especially, by establishing relations between the Frenet Frames in mutual points of two curves several theories have been obtained. The best known of the Bertrand curves discovered by J. Bertrand in 1850 are one of the important and interesting topics of classical special curve theory. A Bertrand curve is defined as a special curve which shares its principal normals with another special curve, called Bertrand mate or Bertrand curve partner. If

$$\alpha^* = \alpha + \lambda N, \lambda = \text{constant}, \quad (1.1)$$

then (α, α^*) are called Bertrand curve pair. If α and α^* Bertrand curves pair, then $\langle T, T^* \rangle = \cos \theta = \text{constant}$ [2, 3]. The definition of n -dimensional Bertrand curves

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