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On hyponormal operators and related classes of operators

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Abstract. There are many classes of operators defined based on operator inequalities. The class of hyponormal operators is perhaps the first such class of operators. An operator T defined on a Hilbert space H is called hyponormal if $T^*T \geq TT^*$, where T^* is the adjoint of the operator T. Since then, many other classes of operators have been introduced, mainly by weakening or modifying the inequality $T^*T \geq TT^*$ in various ways. Among these classes of operators are semi-hyponormal operators, p-hyponormal operators (for $0 < p < 1/2$), log-hyponormal operators , and w-hyponormal operators. Hyponormal operators possess many interesting properties. And the other classes of operators possess the same properties or somewhat weaker properties. In this paper, we will give a review of these classes of operators and their properties.

1. Introduction

Let H be a separable Hilbert Space and let $L(H)$ denote the algebra of bounded linear operators on H. The adjoint operator T^* of an operator T on H is defined by $\langle T^*x, y\rangle = \langle x, Ty\rangle$ for all x and y in H. An operator $T \in L(H)$ is called self-adjoint if $T^* = T$ or equivalently if $\langle Tx, x\rangle$ is real for all $x \in H$. A self-adjoint operator $T \in L(H)$ is called positive semi-definite, denoted $T \geq 0$, if $\langle Tx, x\rangle \geq 0$ for all $x \in H$. An operator $T \in L(H)$ is called normal if $T^*T = TT^*$ (or equivalently $||T^*x|| = ||Tx||$ for all $x \in H$) and hyponormal if $T^*T \geq TT^*$ (or equivalently $||T^*x|| \leq ||Tx||$ for all $x \in H$). An operator $U \in L(H)$ is called unitary if $U^*U = I = UU^*$, where I is the identity operator on H, an isometry if $U^*U = I$ or equivalently, $||Ux|| = ||x||$ for all $x \in H$, and a partial isometry if U^*U is projection from H to a subspace of H. The polar decomposition of an operator $T \in L(H)$ is denoted by $T = U|T|$, where $|T| = (T^*T)^{\frac{1}{2}}$ and U is a partial isometry from $\overline{R(|T|)}$ to $\overline{R(T)}$ such that $N(U) = N(|T|)$ with $\overline{N(\,)}$ and $\overline{R(\,)}$ being the closures of null space and the range, respectively, of an operator. The spectrum of an operator T is defined by $\sigma(T) = \{z \in \mathbb{C} : (T - zI)^{-1}\text{does not exist}\}$ and the spectral radius defined by $r_{sp} = \sup\{|z| : z \in \sigma(T)\}$.

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