Analytic functions concerning with some subordinations

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Abstract. For analytic functions in the class $\mathcal{A}_n$ in the open unit disk $\mathbb{U}$, two subclasses $\mathcal{S}_n^*(\alpha)$ and $\mathcal{K}_n(\alpha)$ of starlike functions and convex functions are introduced. The object of the present paper is to discuss some interesting properties of functions in the classes $\mathcal{S}_n^*(\alpha)$ and $\mathcal{K}_n(\alpha)$ with some subordinations.

1. Introduction

Let $\mathcal{A}_n$ be the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{k=n}^{\infty} a_k z^k \qquad (n = 2, 3, 4, \cdots) \tag{1.1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. If we consider a function $f(z) \in \mathcal{A}_n$ which satisfies

$$\mathrm{Re}\left(\frac{zf'(z)}{f(z)} \right) > 0 \qquad (z \in \mathbb{U}) \tag{1.2}$$

then we say that $f(z)$ is starlike with respect to the origin in $\mathbb{U}$. We denote the subclass of $\mathcal{A}_n$ by $\mathcal{S}_n^*$ consisting of starlike functions in $\mathbb{U}$. Also, we say that $f(z)$ is convex in $\mathbb{U}$ if $f(z) \in \mathcal{A}_n$ satisfies

$$\mathrm{Re}\left(1 + \frac{zf''(z)}{f'(z)} \right) > 0 \qquad (z \in \mathbb{U}). \tag{1.3}$$

This is equivalent to $zf'(z) \in \mathcal{S}_n^*$. We denote by $\mathcal{K}_n$ the subclass of $\mathcal{A}_n$ consisting of all convex functions in $\mathbb{U}$.

Let us consider a function $f(z)$ given by

$$f(z) = z + \frac{1}{n} z^n \qquad (n = 2, 3, 4, \cdots). \tag{1.4}$$

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