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# WAVE COUPLED SYSTEM OF THE $p$-LAPLACIAN TYPE

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***Abstract***. In this work we study the existence of solution and the asymptotic behaviour for a nonlinear coupled system of wave equation of the $p$-Laplacian type given by

$$\begin{cases} u'' - \Delta_p u + |u|^{r-1}u|v|^{r+1} - \Delta u' = 0 & \text{in } Q = \Omega \times (0,T), \\ v'' - \Delta_p v + |v|^{r-1}v|u|^{r+1} - \Delta v' = 0 & \text{in } Q = \Omega \times (0,T), \\ (u(x,0), v(x,0)) = (u_0(x), v_0(x)) & \text{in } \Omega, \\ (u'(x,0), v'(x,0)) = (u_1(x), v_1(x)) & \text{in } \Omega, \\ u(x,t) = v(x,t) = 0 & \text{in } \Sigma = \Gamma \times (0,T), \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded open set with sufficiently smooth boundary $\Gamma$. Existence of solution is made by using Faedo-Gallerkin approximations. We prove the exponential stability when $p = 2$ and polynomial decay if $p > 2$ based on M. Nakao [15, 16].

## 1. Introduction

Throughout this paper, we omit the space variable $x$ of $u(x,t)$ and simply denote $u(x,t)$ by $u(t)$ when there are no conflicts in the notation. C denotes various positive constants depending on the known constants and may be different at each appearance. Let $T > 0$ be a real number, $\Omega \subset \mathbb{R}^n$ be a bounded open set with sufficiently smooth boundary $\Gamma$. We denote by $Q = \Omega \times (0,T)$ the cylinder with lateral boundary $\Sigma = \Gamma \times (0,T)$. Here we consider $2 \le p < \infty$ and $q$ such that $\dfrac{1}{p} + \dfrac{1}{q} = 1$. The duality pairing between the space $W_0^{1,p}(\Omega)$ and its dual $W^{-1,q}(\Omega)$ will be denoted using the form $\langle \cdot, \cdot \rangle$. According to Poincaré's inequality, the standard norm $\| \cdot \|_{W_0^{1,p}(\Omega)}$ is equivalent to the norm $\|\nabla \cdot\|_p$ on $W_0^{1,p}(\Omega)$. Henceforth, we put $\| \cdot \|_{W_0^{1,p}(\Omega)} = \|\nabla \cdot\|_p$. We denote $\| \cdot \|_{L^2(\Omega)} = | \cdot |_2$ and the usual inner product by $(\cdot, \cdot)$. We denote the $p$-Laplacian

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