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HILBERT TRANSFORM OF IRREGULAR WAVE PACKET SYSTEM FOR $L^2(\mathbb{R})$

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Abstract. Let $\{D_{a_j} T_{bk} E_{c_m} \psi\}_{j,k,m \in \mathbb{Z}}$ be an irregular wave packet system and let H be the Hilbert transform on $L^2(\mathbb{R})$. In this paper we give necessary and sufficient conditions for the system $\{D_{a_j} T_{bk} E_{c_m} H\psi\}_{j,k,m \in \mathbb{Z}}$ to be a frame for $L^2(\mathbb{R})$.

1. Introduction and Prelimilaries

A sequence $\{f_k\}$ in a separable Hilbert space $\mathcal{H}$ with inner product $\langle .,. \rangle$ is called a *frame* (or *Hilbert frame*) for $\mathcal{H}$, if there exists finite positive constants A and B such that

$$A\|f\|^2 \leq \|\{\langle f, f_k \rangle\}\|_{\ell^2}^2 \leq B\|f\|^2, \text{ for all } f \in \mathcal{H}. \tag{1.1}$$

The positive constants A and B are called *lower* and *upper* bounds of the frame, respectively. The inequality (1.1) is called the *frame inequality* of the frame. If upper inequality in (1.1) holds, then $\{f_k\}$ is called a *Bessel sequence*. The operator $T : \ell^2 \to \mathcal{H}$ given by

$$T(\{c_k\}) = \sum_{k=1}^{\infty} c_k f_k, \ \{c_k\} \in \ell^2,$$

is called the *synthesis operator* or the *pre-frame operator* of the frame. The adjoint operator $T^* : \mathcal{H} \to \ell^2$ of T is called the *analysis operator* and is given by

$$T^* : f \to \{\langle f, f_k \rangle\}, \ f \in \mathcal{H}.$$

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