

SHARP BOUNDS FOR THE CONVEX COMBINATIONS OF ARITHMETIC, LOGARITHMIC AND GEOMETRIC MEANS IN TERMS OF HARMONIC MEAN

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Abstract. In this paper, we find the greatest values r_1 and r_2 , and the least values s_1 and s_2 in $(0, 1/2)$ such that the double inequalities $H[r_1 a + (1 - r_1)b, r_1 b + (1 - r_1)a] < \alpha A(a, b) + (1 - \alpha)L(a, b) < H[s_1 a + (1 - s_1)b, s_1 b + (1 - s_1)a]$ and $H[r_2 a + (1 - r_2)b, r_2 b + (1 - r_2)a] < \alpha A(a, b) + (1 - \alpha)G(a, b) < H[s_2 a + (1 - s_2)b, s_2 b + (1 - s_2)a]$ hold for all $a, b > 0$ with $a \neq b$ and any $\alpha \in (0, 1)$, where $H(a, b)$, $G(a, b)$, $L(a, b)$ and $A(a, b)$ are the harmonic, geometric, logarithmic and arithmetic means of two positive numbers a and b , respectively.

1. Introduction

For $a, b > 0$ with $a \neq b$, the classical harmonic mean $H(a, b)$, geometric mean $G(a, b)$, arithmetic mean $A(a, b)$, logarithmic mean $L(a, b)$, identric mean $I(a, b)$ are defined by

$$H(a, b) = \frac{2ab}{a+b}, \quad G(a, b) = \sqrt{ab}, \quad A(a, b) = \frac{a+b}{2}, \quad (1.1)$$

$$L(a, b) = \frac{a-b}{\log a - \log b}, \quad I(a, b) = \frac{1}{e} \left(\frac{b^b}{a^a} \right)^{1/(b-a)}, \quad (1.2)$$

respectively. It is well known that the inequalities

$$H(a, b) < G(a, b) < L(a, b) < I(a, b) < A(a, b)$$

hold for all $a, b > 0$ with $a \neq b$.

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