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Some properties of a sequence whose limit is the generalized Ioachimescu's constant

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Abstract. In this short note we have shown some properties of a sequence whose limit is the generalized Ioachimescu's constant. More precise, we show its decreasing monotonicity, concavity, log-concavity, and starshapedness. Our results are simple to be proved but new and with interest. To achieve this we have used on line version of computational knowledge engine Wolfram Alpha.

In 1895 A. G. Ioachimescu [1] proposed a problem regarding to the sequence $(S_n)_{n=1}^{\infty}$ defined by

$$S_n = 1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} - 2\sqrt{n}, \quad n \in \{1, 2, \dots\},$$

who asked to be shown that it is convergent and its limit lies between -2 and -1.

In [2], the author considered the following modification of above sequence, denoted by $(I_n)_{n=1}^{\infty}$,

$$I_n = 1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} - 2(\sqrt{n} - 1), \quad n \in \{1, 2, \dots\}.$$

She has denoted its limit by $\mathcal{J}$ and called it Ioachimescu's constant. Moreover, the same author [3], proved that

$$\frac{1}{2\sqrt{n + \frac{1}{5}}} < I_n - \mathcal{J} < \frac{1}{2\sqrt{n + \frac{1}{6}}}, \quad n \in \{1, 2, \dots\}.$$

Using these inequalities she got that $\mathcal{J} = 0.53964549119....$ In addtion, let us also mention here that the following inequalities pertaining to the sequences $(S_n)_{n=1}^{\infty}$ and $(I_n)_{n=1}^{\infty}$ are given in [5]:

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