DUALS OF $K$-OPERATOR FRAMES IN HILBERT SPACES

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Date of Receiving : 12.04.2017
Date of Revision : 11.11.2017
Date of Acceptance : 10.01.2018

Abstract. We characterize an operator which preserve $K$-operator frame and generate $K$-operator frames from old $K$-operator frames. Also, $K$-dual of a $K$-operator frame is defined and gave some of its characterizations.

1. Introduction

Frames are the redundant system which gives basis like expansion of element of underlying spaces. In 1952 Duffin and Schaeffer [6] introduced frame which gives bases like expansion of a vector in Hilbert spaces. Recall that a countable sequence $\{f_k\}_{k \in \mathbb{N}} \subset \mathcal{H}$ is called frame if there exists positive constants $A$ and $B$ such that

$$A\|f\|^2 \leq \sum_{k \in \mathbb{N}} |\langle f, f_k \rangle|^2 \leq B\|f\|^2, \ f \in \mathcal{H}.$$ 

Since then, various generalizations viz. $g$-frames [13], Fusion frames [1, 3], operator valued frames [8] of frames have been obtained. Găvruta [9] gave the generalization of frames in the form of $K$-frame which has been further generalized by [4] and [12] in the setting of operators. $K$-frames were further studied in [10, 11]. In this paper, we characterize operator which preserve $K$-operator frames and generates new frames from the old ones. We have also obtained some results related to sum of $K$-operator frames. Further, we have defined the notion of $K$-dual of $K$-operator frames. Finally, necessary and sufficient conditions in the form of various results for $K$-dual of $K$-operator frames have been given. This paper is organized as follows: In section 2, we give some basic tools which is needed throughout the paper. In section 3, we prove image of $K$-operator frame under an operator is a $K$-operator frame. Also, we prove that sum of $K$-operator frame and image of $K$-operator frame under an operator is again a $K$-operator frame. In section 4, we define $K$-dual of $K$-operator frame.

2010 Mathematics Subject Classification. 42C15, 42C38.

Key words and phrases. Frames, $K$ operator frames, Bessel sequences.

Communicated by: Laura Gavruta

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