MEROMORPHIC FUNCTIONS AND THEIR DERIVATIVES
CONDITIONALLY SHARE TWO SETS

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Abstract. In the paper, with the aid of relaxed sharing hypothesis, we study the
uniqueness of meromorphic (entire) functions whose derivatives share a finite set.
The results in this paper will improve a number of theorems earlier obtained by
Meng-Hu [11] and Meng [10]. Two examples have been exhibited by us to show that
the conclusions in our results actually occur.

1. Introduction and Definitions

Let us denote by $\mathbb{C} = \mathbb{C} \cup \{\infty\}$. Throughout the paper by a meromorphic function
we shall always mean a meromorphic function in the complex plane $\mathbb{C}$. We adopt the
usual notations of Nevanlinna theory as explained in [8]. By $E$ and $I$ we denote any set
of finite and infinite linear measure respectively of $0 < r < \infty$. For any non-constant
meromorphic function $h(z)$ we define $S(r,h)$ by $S(r,h) = o(T(r,h))$ where $r \rightarrow \infty$, $r \notin E$.

Let $f$ be a non-constant meromorphic function, $a \in \mathbb{C}$ and $p$ be a positive integer.
We denote by $E(a,f)$ the set of zeros of $f(z) - a$ (counting multiplicity) and by $E_p(a,f)$
the set of zeros of $f(z) - a$ with multiplicity $\leq p$ (counting multiplicity).

Let $S \subset \mathbb{C}$. Set

$$E_p(S, f) = \bigcup_{a \in S} E_p(a, f).$$

Then for two non-constant meromorphic functions $f$ and $g$ we say that $f$, $g$ share the set
$S$ truncated $p$ if $E_p(S, f) = E_p(S, g)$. If $p = \infty$, we define $E_p(S, f) = E(S, f) = E_f(S)$.

The inception of set sharing problem in the realm of the theory of meromorphic function was due to the following famous “Gross Question” {see [7]}.

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