

## ESSENTIALLY RATIONALIZED TOEPLITZ HANKEL OPERATORS

RUCHIKA BATRA

Date of Receiving : 20. 03. 2022  
 Date of Revision : 13. 06. 2022  
 Date of Acceptance : 18. 06. 2022

**Abstract.** In this paper, the notion of Essentially Rationalized Toeplitz Hankel Operators on the space  $L^2$  is introduced and some properties about the set of all such kind of operators are discussed, Conditions under which the product of two such operators commute are also discussed. We also discuss essential spectrum of such operators. We define the set ERTHO ( $L^2$ ) as the set of all Essentially Rationalized Toeplitz Hankel operators. Precisely, we attempt to investigate the properties of ERTHO ( $L^2$ ).

### 1. Introduction

The study of Toeplitz and Hankel operators and their generalizations have been a subject of investigation by many researchers around the globe. Toeplitz in the year 1911 introduced the notion of Toeplitz operators and subsequently many researchers like Devinatz [8], Barria and Halmos [7] give the generalization of these operators. Along with Toeplitz operators, Hankel operators were also introduced and become a subject of investigation for the researchers. Hankel operators are the formal companions of Toeplitz operators which have occurred in realization problem for certain discrete time linear systems and in determining which systems are exactly controllable. Motivated by these, Ho in the year 1995 [10] introduced Slant Toeplitz operators on the space  $L^2(\mathbb{T})$ ,  $\mathbb{T}$  being the unit circle in the complex plane. We introduced [2] the notions of Slant Hankel operators and also generalized the notion of Slant Toeplitz operators [2] of  $k^{th}$  order. After that a lot of mathematicians have worked on these kind of operators and also give many kinds of generalizations. The notions of Essentially Toeplitz operators, Essentially Slant Toeplitz operators were also introduced and studied by different mathematicians around the globe.

Ever since the introduction of the class of slant Toeplitz operators, the study has gained importance due to its applications. Villemoes [12] associated the Besov regularity

---

2010 *Mathematics Subject Classification.* 47B35, 47B38.

*Key words and phrases.* Rationalized Toeplitz Hankel Operators, Toeplitz and Hankel Operators.

*Communicated by.* Mayur Puri Goswami