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A GENERALIZED LOCAL FUNCTION IN IDEAL TOPOLOGICAL SPACES

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Abstract. In this paper we introduce a generalization of the notion of local function using operators associated to the topology. Similarly, a notion of compatibility of the ideal with the topology is introduced utilizing this new local function and the classical concept of star closure is extended.

1. Introduction

The notion of closure of a set has been fundamental in the development and expansion of the general topology, to such an extent that it is rare that in some topic of this area a characterization or a result that depends on it is not studied.

In 1979, S. Kasahara [8] introduced the notion of operators associated with a topology; based on this notion D. Jankovic [6] defined the notion of γ-closure, generalizing the notions of θ-closure and δ-closure introduced by N. Veličko [17]. On the other hand, the concept of ideals in topological spaces is treated in the classic text by K. Kuratowski [10]. An ideal $\mathcal{I}$ on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies the following properties: (1) $A\in\mathcal{I}$ and $B\subset A$ implies $B\in\mathcal{I}$; (2) $A\in\mathcal{I}$ and $B\in\mathcal{I}$ implies $A\cup B\in\mathcal{I}$. An ideal topological space (or simply a space) is a topological space (X,τ) with an ideal $\mathcal{I}$ on X and is denoted by $(X,\tau,\mathcal{I})$. For a subset A of X, $A^\star(\mathcal{I},\tau)=\{x\in X : A\cap U\notin\mathcal{I}$ for every $U\in\tau(x)\}$ is called the local function of A with respect to $\mathcal{I}$ and τ [10]. We simply write $A^\star$ in case there is no chance confusion. According to D. Janković and T. Hamlett [7], $Cl^\star(A)=A\cup A^\star$ defines a Kuratowski closure for a topology $\tau^\star(\mathcal{I})$, finer than τ. When there is not chance for confusion, we will simply write $\tau^\star$ for $\tau^\star(\mathcal{I})$.

In addition, for a ideal $\mathcal{I}$ on X, the collection $\beta(\mathcal{I},\tau)=\{U-J : U\in\tau$ and $J\in\mathcal{I}\}$ is a basis for the topology $\tau^\star$. Investigating generalizations of the local function in terms of weak notions of open sets has been the subject of study in many works (see [2, 3, 4, 5, 9, 13, 14, 15]). In this paper, we carry out a detailed study of K. Kuratowski

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