# NONLINEAR DEGENERATE $p$-LAPLACIAN ELLIPTIC EQUATIONS WITH SINGULAR GRADIENT LOWER ORDER TERM 

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Abstract. The present paper aims to study of the Dirichlet problem for a nonlinear degenerate elliptic equation with singular gradient lower order term, we establish existence and regularity estimates for weak solutions of p-Laplacian type elliptic equations of the form

$$
\begin{cases}-\operatorname{div}\left(\frac{|\nabla u|^{p-2} \nabla u}{(1+|u|)^{\gamma}}\right)+\frac{|\nabla u|^{p}}{|u|^{\theta}}=f+u^{r} & \text { in } \Omega \\ u=0 & \text { on } \partial \Omega\end{cases}
$$

where $\Omega$ is a bounded open subset in $\mathbb{R}^{N}, 0<\theta<1, \gamma>0,2<p<N$, $0<r<p-\theta$ and $f$ is a nonnegative function on whose summability we will make different assumptions.

## 1. Introduction and main results

In this article, the problems to be studied are the following

$$
\begin{cases}-\operatorname{div}(a(x, u) \widehat{a}(x, u, \nabla u))+b(x) \frac{|\nabla u|^{p}}{|u|^{\theta}}=\lambda u^{r}+f & \text { in } \Omega,  \tag{1.1}\\ u=0 & \text { on } \partial \Omega,\end{cases}
$$

where $\Omega$ is an open bounded set of $\mathbb{R}^{N}(N \geq 3), B>0, f$ is a positive function belonging to $L^{m}(\Omega)$ with $m \geq 1,0<\theta<1, \lambda \geq 0,0<r<p-\theta$ and $2 \leq p<N$, moreover, assume that $b$ is a measurable function that satisfies a certain condition, where $\nu_{1}$ and $\nu_{2}$ are positive numbers such that

$$
\begin{equation*}
0<\nu_{1} \leq b(x) \leq \nu_{2} \tag{1.2}
\end{equation*}
$$

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