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## NONLINEAR DEGENERATE *p*-LAPLACIAN ELLIPTIC EQUATIONS WITH SINGULAR GRADIENT LOWER ORDER TERM

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Abstract. The present paper aims to study of the Dirichlet problem for a nonlinear degenerate elliptic equation with singular gradient lower order term, we establish existence and regularity estimates for weak solutions of p-Laplacian type elliptic equations of the form

$$\left\{ \begin{array}{ll} -{\rm div}\left(\frac{|\nabla u|^{p-2}\nabla u}{(1+|u|)^{\gamma}}\right)+\frac{|\nabla u|^p}{|u|^{\theta}}=f+u^r & {\rm in}\ \Omega,\\ u=0 & {\rm on}\ \partial\Omega. \end{array} \right.$$

where  $\Omega$  is a bounded open subset in  $\mathbb{R}^N$ ,  $0 < \theta < 1$ ,  $\gamma > 0$ ,  $2 , <math>0 < r < p - \theta$  and f is a nonnegative function on whose summability we will make different assumptions.

## 1. Introduction and main results

In this article, the problems to be studied are the following

$$\begin{cases} -\operatorname{div}\left(a(x,u)\widehat{a}(x,u,\nabla u)\right) + b(x)\frac{|\nabla u|^{p}}{|u|^{\theta}} = \lambda u^{r} + f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$
(1.1)

where  $\Omega$  is an open bounded set of  $\mathbb{R}^N$   $(N \ge 3)$ , B > 0, f is a positive function belonging to  $L^m(\Omega)$  with  $m \ge 1$ ,  $0 < \theta < 1$ ,  $\lambda \ge 0$ ,  $0 < r < p - \theta$  and  $2 \le p < N$ , moreover, assume that b is a measurable function that satisfies a certain condition, where  $\nu_1$  and  $\nu_2$  are positive numbers such that

$$0 < \nu_1 \le b(x) \le \nu_2.$$
 (1.2)

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