Poincare Journal of Analysis & Applications
Vol. 2018 (2(II)), Special Issue (IWWFA-III, Delhi), 77-96
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Matrix-valued wave packet Bessel sequences and symmetric frames in $L^2(\mathbb{R}^d, \mathbb{C}^{s\times r})$

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Date of Receiving : 06.04.2018
Date of Revision : 21.07.2018
Date of Acceptance : 22.07.2018

Abstract. We consider matrix-valued wave packet systems in the matrix-valued function space $L^2(\mathbb{R}^d, \mathbb{C}^{s\times r})$ $(d, s, r \geq 1)$. Some results on matrix-valued wave packet Bessel sequences have been extensively discussed in view to generate frames from Bessel sequences in $L^2(\mathbb{R}^d, \mathbb{C}^{s\times r})$. Necessary conditions for matrix-valued Bessel sequences in terms of an estimate of a series related to the Fourier transform of the matrix-valued wave packet function are given. The frame property of a matrix-valued symmetric wave packet function in higher dimensions is discussed.

1. Introduction

Antolín and Zalik in [1], introduced and studied matrix-valued wavelets for the matrix-valued function space $L^2(\mathbb{R}^d, \mathbb{C}^{n\times n})$. The matrix-valued function space $L^2(\mathbb{R}^d, \mathbb{C}^{n\times n})$ is related to video imaging. Xia and Suter [29] classified and constructed vector-valued (matrix-valued) wavelets with sampling property. They showed that certain linear combinations of known scalar-valued wavelets may yield multiwavelets. Inspired by the work in [1], Jyoti, Deepshikha, Vashisht and Verma [18] studied matrix-valued wave packet frames in $L^2(\mathbb{R}^d, \mathbb{C}^{s\times r})$, where s and r are positive integers. They discussed an interplay between wave packet frames in the Lebesgue space $L^2(\mathbb{R}^d)$ and matrix-valued wave packet frames for the function space $L^2(\mathbb{R}^d, \mathbb{C}^{s\times r})$. The first purpose of this paper is to show some relation between the series related to the Fourier

2010 *Mathematics Subject Classification.* 42C15, 42C30, 42C40, 43A32.
Key words and phrases. Frame, Bessel sequence, symmetric frames, wave packet.
The research of Jyoti is supported by the Council of Scientific & Industrial Research, India (Grant No.: 09/045(1374)/2015-EMR-I). Some part of this paper was prepared at the International Workshop on Wavelets, Frames and Applications (IWWFA)-III (2017), Delhi, during December 14–20, 2017.
We would like to thank the referee for reading the paper carefully and valuable comments towards the improvement of the paper.
Communicated by: Khole Timothy Poumai
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