Retro K-Banach frames in Banach spaces

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Abstract. The notions of K-frame and atomic system in Hilbert spaces were recently introduced by Găvruţa [7]. In this paper, we extend these notions to Banach space setting and defined K-retro Banach frames in Banach spaces and observe that K-retro Banach frame is a generalization of retro Banach frame. Also, we define atomic systems in Banach spaces and proved some results which generalizes some existing results on K-frames. Further, we prove that under certain condition a linear homeomorphic image of a retro K- Banach frame is a retro K- Banach frame. Finally, we study perturbation of retro K-Banach frames and obtain some results.

1. Introduction

Duffin and Schaefer [5] introduced the notion of frames. Let $\mathcal{H}$ be a real (or complex) separable Hilbert space. A countable sequence $\{f_n\} \subset \mathcal{H}$ is called a frame for $\mathcal{H}$, if there exist constants $0 < A \le B < \infty$ such that

$$A\|x\|^2 \le \sum_{i \in \mathbb{N}} \langle x, f_n \rangle^2 \le B\|x\|^2, \quad \text{for all} \quad x \in \mathcal{H}. \tag{1.1}$$

The scalars A and B are called the lower and upper frame bounds of the frame, respectively. They are not unique. The inequality in (1.1) is called the frame inequality of the frame. Feichtinger and Gröcheing [6] extended the notion of frame to Banach space and defined the notion of atomic decomposition. Gröcheing [10] introduced a more general concept for Banach spaces called Banach frame. Casazza, Christensen and Stoeva [2] studied E_d-frames and E_d-Bessel sequences. For more literature on the theory of frames, one may refer to [3, 4, 15]. Retro Banach frames for Banach spaces were introduced and studied by Jain, Kaushik and Vashisht [14]. The notion of K-frame and atomic system for bounded linear operator on Hilbert space was introduced and studied by L. Găvruţa [7]. The notions of K-frame and atomic system for bounded linear operator was extended to Banach spaces setting in [8].

In this paper, we shall study the fusion of retro Banach frames and K-frames and