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A NOTE ON OPERATOR VALUED FRAMES

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Abstract. In this paper, we prove that the Riesz wavelet type basis in $L^2(\mathbb{R}^d)$ is image of an orthonormal wavelet basis under a bounded invertible operator and prove that an orthonormal wavelet basis and an orthonormal wavelet type basis in $L^2(\mathbb{R}^d)$ are unitarily equivalent. Also, it is proved that the image of an orthonormal wavelet basis under unitary operator is an orthonormal wavelet type basis. Further, we prove that dual of Riesz wavelet basis always exists and is biorthogonal to it. Finally, we study dual wavelet frames for a given wavelet frame.

1. Introduction

Wavelet frames were studied considerably by many researchers [1, 2, 3, 4, 6, 8]. Below we give the formal definition of wavelet frame for $L^2(\mathbb{R}^d)$.

Definition 1.1. Let $d \in \mathbb{N}$, $\{\psi^1, \psi^2, ..., \psi^N\} \subseteq L^2(\mathbb{R}^d)$. Define

$$\psi^l_{j,\bar{k}}(\bar{x}) = D_{2^j} T_{\bar{k}} \psi^l(\bar{x}) = 2^{j/2} \psi^l(2^j \bar{x} - \bar{k}),$$

for $\bar{x} \in \mathbb{R}^d, \bar{k} \in \mathbb{Z}^d, j \in \mathbb{Z}$, $l = 1, 2, 3, ..., N$. The system $\{\psi^l_{j,\bar{k}}\}$ is called a *wavelet frame* for $L^2(\mathbb{R}^d)$ if there exist constants $0 < A \leq B < \infty$ such that

$$A\|f\|^2 \leq \sum_{l=1}^{N} \sum_{j\in\mathbb{Z}} \sum_{\bar{k}\in\mathbb{Z}^d} |\langle f, \psi^l_{j,\bar{k}}\rangle|^2 \leq B\|f\|^2, \text{ for all } f \in \mathcal{H}. \tag{1.1}$$

The scalars A and B are called the *lower* and *upper wavelet frame bounds* of the wavelet frame, respectively. They are not unique. If $A = B$, then $\{x_n\}$ is called an *A-tight wavelet frame* and if $A = B = 1$, then $\{x_n\}$ is called a *Parseval wavelet frame.* The inequality in (1.1) is called the *wavelet frame inequality* of the wavelet frame. If $\{\psi^l_{j,\bar{k}}\}$ satisfies the upper frame inequality in (1.1), then $\{\psi^l_{j,\bar{k}}\}$ is called a Bessel wavelet sequence. The system defined in Definition 1.1 is called the homogeneous system and

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