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A NOTE ON FRAMES IN NON-LOCALLY CONVEX BANACH SPACES

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Abstract. Shrinking atomic decompositions in locally convex Banach spaces were studied by Carando and Lassalle [2]. In this paper, we define strongly shrinking atomic decompositions in p-Banach spaces and give necessary and sufficient condition for atomic decomposition to be shrinking and boundedly complete.

1. Introduction

Let X be a vector space over a field $\mathbb{F}$. A p-norm $\|.\|_p$ for $0 < p \leq 1$ on X is a mapping from $X \longrightarrow \mathbb{R}$ satisfying the following properties:

(1) $\|x\|_p \geq 0$, for all $x \in X$.
(2) $\|x\|_p = 0 \iff x = 0$.
(3) $\|\alpha x\|_p = |\alpha|^p \|x\|_p$, for all $x \in X$ and $\alpha \in \mathbb{F}$.
(4) $\|x + y\|_p \leq \|x\|_p + \|y\|_p$, for all $x, y \in X$.

The pair $(X, \|.\|_p)$ is called a p-normed linear space.
If $p = 1$, then the p-norm is equal to norm on X.
A p-normed linear space X over a field $\mathbb{F}$ is called a p-Banach space if it is complete.
A linear operator $T : (X, \|.\|_p) \longrightarrow (Y, \|.\|_q)$ is said to be bounded if there exists a real number $M > 0$ such that $\|T(x)\|_q^{\frac{1}{q}} \leq M \|x\|_p^{\frac{1}{p}}$, for all $x \in X$.

The collection of all bounded linear operators from the p-Banach space X to the q-Banach space Y is denoted by $B(X, Y)$ which is a Banach space with norm given by

$$\|T\| = \sup_{\substack{x \in X \\ x \neq 0}} \frac{\|T(x)\|_q^{\frac{1}{q}}}{\|x\|_p^{\frac{1}{p}}}.$$

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