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THE ERROR CONVERGENCE OF THE WEAK GALERKIN METHOD FOR TWO-DIMENSIONAL NON-LINEAR CONVECTION-DIFFUSION PROBLEM

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Abstract . This paper presents the fully-discrete scheme for the solution of two-dimensional non-linear convection diffusion equations by using the Crank-Nicolson-Weak Galerkin finite element methods. We introduce and analyze stability. The error estimate and an optimal order of (L^2 and H^1)-norm are proved. We confirm the theoretical results with some numerical examples.

1. Introduction

The examination of convection-diffusion problems has piqued the interest of a growing number of specialists, engineers, and mathematicians. Due to challenges with proper resolution of the so-called boundary layers, special emphasis is dedicated to problems with convection dominating diffusion. Upwinding or artificial viscosity are commonly used in efficient finite difference or finite element methods. We consider the following the Dirichlet problem for non-linear convection-diffusion, which seeks an unknown function $u = u(x,t)$ satisfying [12]:

$$\frac{\partial u}{\partial t} - \nabla \cdot (a(u)\nabla u) + b(\vec{u}) \cdot \nabla u + cu = f; \quad \Omega \times (0,T], \tag{1.1}$$

$$u(x,t) = g; \quad \Gamma \times (0,T], \tag{1.2}$$

$$u(x,0) = u^0(x); \quad x \in \overline{\Omega}, \tag{1.3}$$

in a domain $\Omega \subset R^2$ with boundary Γ, where $x = (x_1, x_2)$ and $f = f(x,t) \in \Omega \times (0,T]$, and the diffusion coefficient and nonlinear function are denoted by $a(u)$, and the convection coefficient and nonlinear function are denoted by $b(u)$, and $c = c(x)$ is a

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