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EXISTENCE AND UNIQUENESS THEOREMS FOR NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. We consider nonlinear partial differential equations of Fuchs type, establishing the existence and uniqueness of the solutions to these equations when the nonlinearities are continuous with respect to the Fuchsian variable and holomorphic with respect to the other variables. The method of proof is based on the application of the fixed point theorem in some Banach spaces defined by majorant functions.

1. Introduction

In [7], the author proved the existence and uniqueness of a solution of a nonlinear Fuchsian partial differential equation. This solution was established in the space of holomorphic functions with respect to the Fuchsian variable and in Gevrey spaces with respect to the other variables.

The present work aims to weaken the regularity hypothesis on the Fuchsian variable of [7]. We consider a nonlinearity $f(t, x, y)$ which is continuous with respect to the Fuchsian variable t and (complex) analytic in the other variables. We restrict our study to one Fuchsian variable and consider operators whose principal parts are with non-constant coefficients, hence, in a more general form than those of [7].

The search for the existence and uniqueness of holomorphic solutions for Fuchsian equations has been studied in the works of N. S. Madi and M. Yoshino [10], M. Miyake [11], and P. Pongérard [12]. The search for the explicit representation of holomorphic solutions for these Fuchsian equations has been studied for example in [4]. However, as was mentioned in [1], the results for this type of problems where the nonlinearity $f(t, x, y)$ is assumed to be continuous in t are rare. We cite the work of Baouendi-Goulaouic [3], where $f(t, x, y)$ is smooth with respect to t (but not necessarily analytic) and analytic with respect to the other variables. In [13], where f is smooth in t (not

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