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## A STUDY OF EXTRAPOLATION PROPERTIES OF $C_0$ -SEMIGROUPS IN NON-COMMUTATIVE SETTING

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**Abstract.** The semigroup defined on  $\mathcal{L}^2(\mathcal{X})$  can be extrapolated to  $\mathcal{L}^p(\mathcal{X})$ , preserving the properties of the original semigroup. In this paper, we extend this framework to the non-commutative setting.

## 1. Introduction

Consider a consistent family of semigroups  $U_p$  defined on  $\mathcal{L}^p(\mathcal{X})$  for  $1 \leq p < \infty$ . Suppose that the semigroup  $U_{p_0}$  satisfies a specific property for some  $p_0$ . A key question is whether this property can be extrapolated to all (or some)  $1 \leq p < \infty$ , meaning that  $U_p$  automatically inherits the same property for these values of p. This question is particularly relevant when p = 2, as  $\mathcal{L}^2(\mathcal{X})$  is a Hilbert space, allowing one to leverage the robust and well-established Hilbert space theory. These extrapolation results facilitate transferring the same properties to  $\mathcal{L}^p$ -spaces, which are often more challenging to study directly.

In classical semigroup theory, several properties (see [2]), such as the  $C_0$ -property, holomorphy, bounded generator, norm continuity, compact resolvent, exponential stability, strong stability, positivity, irreducibility, and spectral properties of the semigroup U defined on  $\mathcal{L}^2(\mathcal{X})$ , are preserved by the extrapolated semigroups  $U_p$  on  $\mathcal{L}^p(\mathcal{X})$  for  $1 \leq p < \infty$ . However, these properties cannot always be directly generalized to the extrapolated semigroups  $U_p$ . In this article, we extend these properties to semigroups defined on non-commutative  $\mathcal{L}^p$ -spaces.

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