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A NOTE ON THE COMPARISON BETWEEN THE HENSTOCK AND BOCHNER INTEGRALS FOR NON-NEGATIVE FUNCTIONS WITH VALUES IN AN ORDERED BANACH SPACE

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Abstract. A well-known result for real-valued functions states that every non-negative Henstock-Kurzweil integrable function is Lebesgue integrable. However, this result does not hold when considering the Henstock and Bochner integrals for functions taking values in an ordered Banach space. In this work, we provide sufficient conditions under which a non-negative Henstock integrable function is also Bochner integrable.

1. Introduction

In 1904, Lebesgue introduced a theory of integration for real-valued functions that addresses some limitations of Riemann's integration theory. However, the Lebesgue integral cannot integrate all derivatives. In the late 1950s, Kurzweil and Henstock independently developed integration theories for real-valued functions that fixed this issue. These two theories were later shown to be equivalent and are now collectively referred to as the Henstock-Kurzweil integral. It is well known that the space of Lebesgue integrable functions is properly contained in the space of Henstock-Kurzweil integrable functions, demonstrating that the Henstock-Kurzweil integral is a generalization of the Lebesgue integral [6].

There are several generalizations of the Lebesgue integral for functions taking values in a Banach space. For example the Bochner, Pettis, and Dunford integrals. Cao [3] generalized the Henstock and Kurzweil integrals for functions taking values in a Banach

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