

UNIQUENESS AND SOLVABILITY OF THE SINGULAR CAUCHY PROBLEM IN GEVREY SPACES

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Abstract. The present work aims to generalize the global results of [7] where the coefficients of the Fuchsian principal part are assumed to be constant. We consider a more general Fuchsian operator whose coefficients of the Fuchsian principal part are of Gevrey classes. We prove the global existence and uniqueness of a solution in the space of sufficiently differentiable functions with respect to the Fuchsian variable and in Gevrey spaces with respect to the other variables. Contrary to [7] where the study is restricted to projective Gevrey spaces, we consider both projective and classical Gevrey spaces.

1. Introduction

We study in this article the Cauchy problem for partial differential equations of Fuchs type in the sense of Baouendi and Goulaouic [1] by using the fixed point method by the concept of majorant functions. Utilizing this method, Pongérard [16] proved the existence and uniqueness of a global entire solution of the Fuchsian Cauchy problem where the coefficients of the Fuchsian principal part are constants. His results are a generalization of those obtained in [14] for the non characteristic Cauchy problem to the case of Fuchsian operators.

In [5], the authors generalized the global results of [16] to the Fuchsian Goursat problem with constant principal part, and they simplified the proof of the local existence and uniqueness of a solution to the holomorphic Goursat problem with non constant (holomorphic) principal part.

In [4], the authors generalized the results of [16] to the space of holomorphic functions with respect to the Fuchsian variable and to classical Gevrey spaces $G^{\{d\}}$ as well as to projective Gevrey spaces $G^{(d)}$ with respect to the other variables.

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