

CHARACTERIZATION OF CONTRACTIVE PROJECTIONS IN TERMS OF CONDITIONAL EXPECTATION OPERATORS IN GENERALIZED VARIABLE EXPONENT LEBESGUE SPACES

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ABSTRACT. Let (E, Σ, μ) be a σ -finite measure space, A be a Banach lattice algebra possessed the Radon-Nikodym property, and $L^{p(\cdot)}(E, \Sigma, \mu; A)$, $p \in P^{\log}(E)$, $1 < p_m, p_S < \infty$ be variable exponent Lebesgue space of all A valued measurable functions equipped with the Luxemburg norm. A linear operator $P : L^{p(\cdot)}(E, \Sigma, \mu; A) \rightarrow L^{p(\cdot)}(E, \Sigma, \mu; A)$ such that $P = P^2$ and $\|P\|_{L^{p(\cdot)} \rightarrow L^{p(\cdot)}} \leq 1$ is called contractive projection. We establish a description of the range of a positive contractive projection in terms of conditional expectation. To prove our main result, we generalize Levi's lemma to the Banach lattice case and apply integral representation theory for bounded linear operators in variable exponent Lebesgue spaces of lattice algebra functions.

1. Introduction

Let E be a locally compact space and let (E, Σ, μ) be a σ -finite, complete measure space. Let X be a Banach space. Let the set function $\chi : \Sigma \rightarrow X$ be a μ -absolutely continuous measure of bounded variation, then the Banach space X is said to possess the Radon-Nikodym-Bochner property if there exists a μ -Bochner integrable function $\omega : E \rightarrow X$ such that $\chi(D) = \int_D \omega d\mu$, for all $D \in \Sigma$. S. Moedomo and J. J. Uhl [9] showed that if a Banach space X is reflexive and a μ -continuous measure $\omega : E \rightarrow X$ has finite variation then a range of ω is separable then the Radon-Nikodym theorem holds for X or the Banach space X has Radon-Nikodym property [3, 9, 11].

Now, let Ξ be a sub-ring of Σ with maximal element E_0 then there the conditional expectation operator E_{Ξ} prescribes to an integrable function ϕ the Radon-Nikodym derivative of $\int_{E \cap E_0} \phi d\mu$ with respect to μ restrained to Ξ . The classical Ando-Douglas

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