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SOME RESULTS ON FRAMES IN BANACH SPACES

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Abstract. In this article, the notion of equivalent Schauder frame is defined and studied. A necessary condition for a Banach space having an equivalent Schauder frame is given. Also, the relationship between Schauder frame for a Banach space and its complemented subspace is investigated. Finally, it is proved that if a Banach space $\mathcal{B}$ has an exact Banach frame and $\mathcal{B}_1$ is any other Banach space, then the space of all continuous linear mappings of $\mathcal{B}_1$ into $\mathcal{B}$ is isomorphic to some sequence space.

1. Introduction and Preliminaries

While working with families of exponentials $\{e^{i\lambda_n t}\}_{n\in\mathbb{Z}}$, Duffin and Schaeffer [6], defined the notion of frame as follows:

A sequence $\{x_n\}$ in a separable Hilbert space $\mathbb{H}$ (with inner product $\langle\cdot,\cdot\rangle$) is said to be a Hilbert frame for $\mathbb{H}$, if one can find positive constants $\mathbb{A}$ and $\mathbb{B}$ such that

$$\mathbb{A}\|x\|^2 \leq \sum_n |\langle x, x_n\rangle|^2 \leq \mathbb{B}\|x\|^2, \quad \text{for all } \ x \in H. \tag{1.1}$$

The positive constants $\mathbb{A}$ and $\mathbb{B}$, respectively, are called *lower* and *upper frame bound* for the frame $\{x_n\}$ and collectively known as *frame bounds* for the frame $\{x_n\}$. These bounds may not be unique. The inequality (1.1) is termed as the *frame inequality* for the frame $\{x_n\}$.

A frame $\{x_n\}$ is known as a *tight frame* if it is possible to choose $\mathbb{A} = \mathbb{B}$ as frame bounds and is called as the *Parseval frame* if it is possible to choose $\mathbb{A} = \mathbb{B} = 1$. Also, a frame $\{x_n\}$ is termed as an *exact frame* if the withdrawal of one element say x_i renders the remaining elements $\{x_n\}_{n\neq i}$ no more a frame for H. The concept of a Banach frame in a Banach space was defined by Gröcheing [7] as follows:
Let $\mathcal{B}$ be a Banach space over the scalar field $\mathbb{K}$ ($\mathbb{R}$ or $\mathbb{C}$). Using $\mathbb{N}$ as an index, let $\mathcal{B}_d$ be

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