RESOLVENT OF A LINEAR RELATION: INVARIANT SUBSPACES AND GROUP INVERTIBILITY

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Abstract. This article studies some properties of $R-$invariant subspace of a closed linear relation on a Banach space. In addition, some results on the connection between the $R-$invariant subspaces for linear relations and their inverse relation are presented. The group invertibility and the connection that exists with infinity as an isolated point of the spectrum of the linear relation are discussed.

1. Introduction

The generalized inverses of linear operators are very useful tools in mathematical applications, mainly in differential equations and mathematical models, where a generalized inverse is one of the most used. Recently, the concept of Drazin inverse and group inverse has been introduced for the linear relation, more results can be found in [2], [4] and [5] .

Any linear subspace $\widehat{A}$ of $X \times Y$, where X and Y are complex Banach spaces, is called a linear relation between the spaces X and Y. It is easy to see that every linear relation is identified with the map $A : X \to 2^Y$, $x \mapsto Ax := \{y \in Y : (x, y) \in \widehat{A}\}$. We call $\widehat{A}$ the graph of the map A and denote $Gr(A) := \widehat{A}$. The domain and the image of a linear relation A are defined by

$$Dom(A) := \{x \in X : Ax \neq \varnothing\} \text{ and } ImA := \{y \in Y : \exists\, x \in X, y \in Ax\},$$

respectively. The inverse linear relation of A is defined by its graph as $Gr(A^{-1}) := \{(y, x) \in Y \times X : (x, y) \in Gr(A)\}$. Other sets of utmost importance are the multivalues of A, defined by $A(0) := \{y \in Y : (0, y) \in Gr(A)\}$; the kernel of A, defined by $KerA := A^{-1}(0)$ (the kernel of A is the multivalues of A^{-1}). It is clear that $DomA$ and $KerA$ are subspaces of X, ImA and $A(0)$ are subspaces of Y. If $Gr(A)$ is a closed subspace

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