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ON THE RESIDUAL JULIA SET OF SEMIGROUPS GENERATED BY TRANSCENDENTAL ENTIRE FUNCTIONS

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Abstract. The motivation behind this paper is to extend some results regarding the residual Julia set of transcendental entire functions to the more general setting of semigroups generated by transcendental entire functions. We define the residual Julia set for transcendental semigroups and provide a few examples of semigroups for which the residual Julia set is non-empty. Some elementary properties of this set are discussed. It is shown that the Julia set of a transcendental semigroup is the closure of the union of the residual Julia sets of its elements. We give certain results related to the existence of the residual Julia set and the connectivity of the Fatou set and the Julia set. Finally, some results about the existence of buried points in the Julia set are given.

1. Introduction

Let $f : \mathbb{C} \to \mathbb{C}$ be a transcendental entire function and $\{f^n = f \circ f^{n-1}\}_{n\in\mathbb{N}}$ be the iterative sequence of f, taking f^0 as the identity function. The Fatou set $F(f)$, is the collection of points for which $\{f^n\}$ is well-defined and normal in some neighbourhood of that point. The complement of the Fatou set is the Julia set, denoted by $J(f)$. If a family of holomorphic functions on a domain D omits the same 2 values in $\mathbb{C}$, then the family is normal in D. This is known as the Fundamental Normality Test. The escaping set of f, $I(f) = \{z \in \mathbb{C} : f^n(z) \to \infty \text{ as } n \to \infty\}$, was introduced by A. E. Eremenko [6]. It is known from the same paper that for a transcendental entire function f, $J(f) = \partial I(f)$ and all components of $\overline{I(f)}$ are unbounded, where ∂A is the boundary of A and $\overline{A}$ is the closure of A. Rippon and Stallard defined a connected structure called an infinite spider's web in [15] and discussed the presence of spider's web structure in

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