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RENORMALIZED AND ENTROPY SOLUTIONS TO NONLINEAR ANISOTROPIC DEGENERATE PARABOLIC PROBLEMS WITH VARIABLE EXPONENTS AND L^1 DATA

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Abstract. In this paper, we prove the existence of entropy and renormalized solutions for a specific type of nonlinear anisotropic parabolic equations in the cylinder $(0,T)\times\Omega$. The growth conditions are defined by $p_i(x)$, and the data is in L^1. The functional setting involves Lebesgue-Sobolev spaces with variable exponents. Our results extend the corresponding findings in the constant exponent isotropic and anisotropic cases and some results presented in C. Zhang et al. (J. Differential Equations 248 (2010), 1376–1400).

1. Introduction

We want to solve the following anisotropic parabolic problem:

$$(P)\qquad \begin{cases} \partial_t u + Au + F(t,x,u) &= f \quad \text{in } Q_T=(0,T)\times\Omega \\ u(0,x) &= u_0 \quad \text{in } \Omega \\ u(t,x) &= 0 \quad \text{on } \Gamma_T=(0,T)\times\partial\Omega, \end{cases}$$

where $T>0$ is a real number, $f\in L^1(Q_T)$, $u_0\in L^1(\Omega)$, Ω is a bounded open subset of $\mathbb{R}^N$ $(N>2)$ with boundary denoted by $\partial\Omega$ and A is the operator given by

$$Au = -\sum_{i=1}^{N} D_i(d_i(t,x,u)|D_iu|^{p_i(x)-2}D_iu).$$

Here, we suppose that $d_i:(0,T)\times\mathbb{R}^N\times\mathbb{R}\to\mathbb{R}$, $\quad i=1,...,N$ are Carathéodory functions and satisfying:

$$\frac{c_2}{(1+|u|)^{\varrho}} \le d_i(t,x,u) \le c_1, \tag{1.1}$$

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