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DENSE AND SUBSPACE DENSE SUBSETS IN FINITE-DIMENSIONAL SPACES

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Abstract. This note is motivated by the article of Bamerni, Kadets and Kiliçman [J. Math. Anal. Appl. 435 (2), 1812–1815 (2016)]. We consider the remaining problem which claims that if A is a dense subset of a finite dimensional space X, then there is a non-trivial subspace M of X such that $A \cap M$ is dense in M. We show that the above problem has a negative answer when $X = \mathbb{K}^n$ ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) for every $n \geq 2$.

1. Introduction

In [1, Theorem 2.1], Bamerni, Kadets and Kiliçman established that if A is a dense subset of a Banach space X, then there is a non-trivial closed subspace M of X such that $A \cap M$ is dense in M. By a non-trivial subspace M of X, we mean that M is non-zero and distinct from X. We acknowledge here that the authors of [1] do not clearly mean X of infinite dimension. So Theorem 2.1 in finite dimension remains an open problem.

Problem 1. Is Theorem 2.1 of [1] true for X of finite dimension?

In the present note, we show that Problem 1 does not hold in finite dimension, that is [1, Theorem 2.1] is not true for every finite dimensional space $X = \mathbb{K}^n$, $n \geq 2$, $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$.

In the sequel, $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$ denote the sets of non-negative integers, integers and rational numbers, respectively, while $\mathbb{N}_0$ and $\mathbb{R}_+^*$ denote the set of positive integers and positive real numbers, respectively. For a row vector $v \in \mathbb{K}^n$, we will denote its transpose by v^T. If $v_1, \dots, v_p \in \mathbb{K}^n$, $p \geq 1$, we denote by $\operatorname{span}\{v_1, \dots, v_p\}$ the vector subspace of $\mathbb{K}^n$ generated by $v_1, \dots, v_p$. A subset $E \subset \mathbb{K}^n$ is called *dense* in $\mathbb{K}^n$ if $\overline{E} = \mathbb{K}^n$, where $\overline{E}$ denotes the closure of E. It is called *nowhere dense* if $\overline{E}$ has empty interior.

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