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IMPROPER INTEGRALS IN TOPOLOGICAL MEASURE SPACES

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Abstract. Many generalized integrals, that include improper ones, have been introduced in the literature to solve important problems. For example the Denjoy, Perron, and Henstock–Kurzweil integrals, each of which has its corresponding theoretical development. This leads us to the search for a particular type of improper integration within the classical framework of measure theory, which, while it may not fully resolve the problems that led to the development of the mentioned integrals, can nevertheless be applied to most practical cases and allow us to address them directly using the well-known techniques of measure theory, without the need for the development of new constructions.

In this paper, we select and provide the foundations for a particular method of improper integration that can achieve the objectives of the search described. For the sake of convenience in terminology, we refer to this method as J-integration. We then demonstrate its vectorial structure and show how it can be applied to different practical problems.

1. Introduction

Given a measure space, traditional definitions of improper integrals have been taken historically as limits of integrals over increasing sequences of measurable sets that converge to the integration domain almost everywhere. This approach includes, for example, integrals of unbounded functions, of functions defined on unbounded intervals and the classical definition of convergence of the series, where the counting measure is considered. In [4], Jiménez presents an approach to improper integration, which includes some of these traditional ideas, as described below.

In what follows, we exclude trivial cases. If μ is a positive σ-finite measure, then there exists an infinite increasing sequence $(A_n \uparrow)$ of measurable sets and a μ-negligible

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