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# DYNAMIC INTERACTIONS IN A CAPUTO FRACTIONAL-ORDER SIR MODEL WITH SATURATED INCIDENCE AND TIME DELAYS

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***Abstract***. This study develops a fractional-order SIR-type pandemic system utilizing the Caputo derivative, exploring its complex dynamics and elucidating the significant role of the reproduction number in the model. A comprehensive explanation of its exact calculation and relevance is provided. An in-depth mathematical analysis clarifies critical aspects of the system behavior, focusing on both endemic and infection-free states. The investigation delves into the model's overall behavior, with a detailed examination of the existence of both backward and Hopf bifurcations. The effects of various model parameters on the system dynamics are identified, highlighting their complex interactions. Additionally, the research expands to include a time delay component, acknowledging its relevance in modelling real-world scenarios. By incorporating temporal factors such as incubation periods and delays in treatment efficacy, this study enhances the understanding of disease dynamics. An optimal control problem is defined, and its impact on disease dynamics is explored through the integration of a time-dependent therapy control variable. Analytical results are validated through numerical simulations utilizing the predictor-corrector technique, confirming the stability of the model. By incorporating a temporal delay component, this research offers a thorough examination of a Caputo derivative-based fractional-order SIR-type pandemic model, providing valuable insights for the development of effective disease management strategies.

## 1. Introduction

Human lifestyles driven by scientific and technological advancements, the spectre of infectious diseases remains a formidable challenge to civil societies worldwide. Despite the removal of numerous barriers facilitated by scientific progress, infectious diseases