

## **$\epsilon$ -CONVEXITY AND HADAMARD-TYPE INEQUALITIES: A NOVEL APPROACH**

NIKHIL KHANNA, TARACHAND PRAJAPATI<sup>†</sup>, AND SILVESTRU SEVER DRAGOMIR

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**Abstract.** In this paper, we investigate the concept of  $\epsilon$ -convex functions and establish new integral inequalities, particularly extending Hadamard's inequality for this class of functions. We define  $\epsilon$ -convexity and explore its fundamental properties, demonstrating that it forms a positive cone and is preserved under function composition. Furthermore, we derive new estimates that enhance Hermite-Hadamard-type inequalities for differentiable functions whose derivatives exhibit  $\epsilon$ -convexity. Our findings contribute to the broader study of convex analysis and its applications in mathematical inequalities.

### **1. Introduction**

The concept of convexity plays a fundamental role in the development of mathematical inequalities, particularly in the study of integral inequalities, matrix inequalities, and operator inequalities. Over the years, convex functions have been widely employed to derive stronger and more refined inequalities, serving as essential tools in mathematical analysis and optimization theory [4]. Convex functions are at the core of numerous classical inequalities. For example, Jensen's inequality, a fundamental result in convex analysis, extends to various contexts, including affine combinations and multi-variable settings [16]. Additionally, the Hermite-Hadamard inequality, which provides bounds for convex functions over an interval, has been generalized through new approaches, such as harmonic convexity and fractional convexity [21]. These developments highlight the growing importance of convexity in refining classical mathematical inequalities.

Recent studies have introduced novel convexity-based inequalities that provide sharper bounds than traditional approaches. For instance, radical convexity has been

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<sup>†</sup> *Corresponding author*