

GENERALIZED KURZWEIL-STIELTJES INTEGRALS

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Abstract. Motivated by Malý and Kuncová (2019), we study a scale of generalized Kurzweil-Stieltjes integrals on the real line. Our first generalization is based on the concept of p -oscillation instead of ordinary oscillation, which was the key concept in their definition. Since Kurzweil-Stieltjes integral can be equivalently defined using ordinary oscillation, our improvement consists in using p -oscillation instead of ordinary one and α -systems instead of partitions of the interval. These changes lead to a larger classes of integrable functions. The key concepts like p -oscillation and p -median are discussed in detail. The integrals introduced are non-absolutely convergent and cover both the Lebesgue and Henstock-Kurzweil integral. Our main results concern e.g. the uniqueness of the indefinite generalized integrals and the dependence of the classes of integrable functions on the parameters $p \in [1, \infty]$ and $\alpha \geq 1$. At the end, an analogue of the known Hake's theorem is presented.

1. Introduction

The main topic of this paper is study of the class of HKS_α^p integrals based on minimization of sums of p -oscillations instead of ordinary oscillations and α -systems instead of partitions. Our main results are contained in the closing Section 4.

Throughout the paper, we assume that I denotes a bounded interval. In Section 2, we introduce the definition of p -median of a measurable function as a minimizer of its L^p -norm which is the key concept in the definition of p -oscillation and generalized Kurzweil integral. However, it is not obvious how to compute p -oscillation of a given function. It leads us to question if it is possible to classify p -medians of a given function for any p , i.e. how to find a number $c(p) \in \mathbb{R}$ for given measurable function $f \in L^p$ such that the value of the norm $\|f - c(p)\|_p$ will be minimal. We can answer this question for $p = 1$, $p = 2$ and $p = \infty$. In particular, the p -median for $p = 1$ is the classic median, for

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