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CONTROLLABILITY OF MEASURE DRIVEN INTEGRODIFFERENTIAL EVOLUTION SYSTEMS WITH STATE-DEPENDENT DELAY AND NONLOCAL CONDITIONS

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Abstract. This study examines a class of integrodifferential measure evolution systems featuring state-dependent delays and nonlocal conditions and investigates the existence of mild solutions and exact controllability for these systems. Initially, we demonstrate that the problem at hand has mild solutions. Afterwards, we investigate exact controllability by applying the theory of the resolvent operator, the Kuratowski measure of noncompactness, and the Mönch fixed point theorem, all without requiring Lipschitz continuity on the nonlinear term. We conclude by providing an example that demonstrates the validity of the study.

1. Introduction

This work examines the exact controllability of the following nonlocal delayed integrodifferential equation:

$$\begin{cases} v'(t) = Av(t) + \displaystyle\int_0^t \Phi(t-s)v(s)\mathrm{d}s + [\mathcal{C}u(t) + g(t, v_{\delta(t,v_t)})]\mathrm{d}\mu(t), \quad t \in J, \\ v(t) + \Xi(v_{t_1}, v_{t_2}, \cdots, v_{t_m})(t) = \psi(t),\ t \in (-\infty, 0], \end{cases} \tag{1.1}$$

where $J = [0, b], 0 < t_1 < t_2 < \cdots < t_{m-1} < t_m \leq b;\ m \in \mathbb{N},\ \psi \in \mathfrak{B}_\tau$, with $\mathfrak{B}_\tau$ a phase space which will be specified later in the preliminary's section. The state $v(\cdot)$ of the system (1.1) takes values in the Banach space $\mathbb{X}$. The operator A of domain $D(A) \subseteq \mathbb{X}$ is the infinitesimal generator of c_0-semigroup $\{T(t)\}_{t\geq 0}$; $(\Phi(t))_{t\geq 0}$ is closed linear operators on $\mathbb{X}$ with domain $D(\Phi(t)) \supset D(A)$ which is independent of t. The nonlinear function $g : J \times \mathfrak{B}_\tau \to \mathbb{X}$ will be described later. The history function $v_t : (-\infty, 0] \to \mathbb{X}$ is an element of $\mathfrak{B}_\tau$ and is defined by $v_t(\varrho) = v(t + \varrho), \varrho \in (-\infty, 0]$. Moreover, the nonolocal

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