

## NUMERICAL CONSTRUCTION OF OPTIMIZED RBF SHAPE PARAMETER

GEETA ARORA AND KIRAN BALA<sup>†</sup>

Date of Receiving : 10. 11. 2024  
Date of Revision : 02. 07. 2025  
Date of Acceptance : 10. 08. 2025

**Abstract.** The present article emphasizes the optimization techniques integrated with the Radial Basis Function (RBF) for numerical solutions of a nonlinear partial differential equation. Optimization techniques have been useful in determining the parameters required to solve various mathematical models. The numerical treatment of a partial differential equation using RBF requires the optimized shape parameter, which is responsible for defining the shapes of RBFs with accurate results. This article proposes a novel approach for determining the shape parameter of RBF, which is crucial for identifying the numerical solutions of the Fitzhugh-Nagumo (FN) equation. This proposed approach is a hybrid form of the mesh free RBF pseudo-spectral (RBF-PS) method and particle swarm optimization (PSO) technique. The numerical solution obtained for the two problems of the FN equation at different number of node points highlights the efficacy of the proposed hybrid method with rate of convergence. The results obtained in the form of  $L_\infty$ ,  $L_2$ , RMS error norms are presented in figures and tables that are in good agreement with the available results in the literature.

### 1. Introduction

In modern research, numerical treatment of the mathematical model based on optimization techniques plays a vital role. Optimization has numerous applications across various fields, including engineering, economics, finance, operations research, and more. Optimizing the shape parameter has become the main concern for researchers since RBFs came into existence for finding solutions to various mathematical models. PSO is one of the population-based meta-heuristic optimization techniques [16], as well as an evolutionary algorithm for finding solutions to optimization problems in a wider range of applications. PSO finds its applications in a wide range of fields such

---

2010 *Mathematics Subject Classification.* 65M70, 97N40.

*Key words and phrases.* Particle Swarm Optimization, Fitzhugh-Nagumo Equation, Pseudo-spectral Method, Shape parameter, Radial Basis Function.

*Communicated by:* Oleg V. Kravchenko

<sup>†</sup> *Corresponding author*