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DEVELOPING A SEQUENTIALLY CONNECTED BÉZIER CURVE METHOD TO IMPLEMENT DERIVATIVE BEHAVIORS

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Abstract. In mechanics or computer graphics, derivative discontinuities are used to model abrupt physical or structural transitions. This study presents a generalizable method for constructing functions that are differentiable at every point, free of sharp corners, yet possess derivatives that not only change sign from negative to positive at an extreme point but are also discontinuous there. The core idea of our approach is to sequentially connect concave and convex quadratic Bézier curves in such a way that the derivative becomes discontinuous at an extreme point, while keeping the entire construction confined between smooth reference curves. In contrast, by analyzing the properties of convex functions, we show that such discontinuities in the derivative cannot occur within the convex setting.

1. Introduction

Differential discontinuity properties at an extreme point of a curve that appears smooth overall are important for capturing abrupt but natural transitions of continuous motions, such as sharp changes in velocity or potential in mechanical systems, crease lines in computer-aided graphical models. Thus, constructing appropriate functions with discontinuous derivative at an extreme point are essential in modeling in various scientific and engineering fields, including mechanics, computer graphics, and control theory. For example, various sliding mode control methods exploit the differential discontinuity of sliding variables to reduce the chattering (high-frequency amplitude vibration) problem of multi-input multi-output mechanical systems [7]. And the differential discontinuities, used in the concept of semi-smooth creases [9], are important in computer grphics for achieving realistic edges while minimizing memory usage [17]. In the constructions of those phase functions, differentiability is required to maintain physical or geometric

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