

MULTI-WINDOW WILSON SYSTEM IN $L^2(\mathbb{R}^d)$

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Abstract. This study aims to introduce the multi-window Wilson system in $L^2(\mathbb{R}^d)$. The multi-window Wilson system is characterized in terms of the multi-window Gabor system, and based upon this characterization, the necessary and sufficient conditions for the multi-window Wilson system to be a multi-window Wilson frame in $L^2(\mathbb{R}^d)$ are obtained. Furthermore, the sufficient conditions are given for the multi-window Wilson system, such that it makes a multi-window Wilson Bessel sequence in $L^2(\mathbb{R}^d)$. The necessary and sufficient conditions for examining the stability of the multi-window Wilson frame are obtained and illustrated with examples and counterexamples.

1. Introduction

In today's interconnected world, signals are a fundamental aspect of communication, forming the backbone of data transmission and processing. Effective analysis of signals is essential for advancements in communication theory, which seeks to ensure reliable information exchange. Among all the methods developed for analyzing signal properties, a key challenge in signal processing is representing a signal as a combination of simpler, more fundamental components or atoms. The concept of decomposing a signal using translation and modulation operators into elementary components was introduced by Gabor [12] in 1946. But this decomposition was only limited to the localized content of a signal. In 1952, Duffin and Schaeffer [10] developed the idea of frames for Hilbert spaces to examine certain profound difficulties in non-harmonic Fourier series. Although the frames are more flexible, this idea did not generate much interest. After three decades, the concept of frame was reintroduced by Young [26] in an abstract setting. Later, Daubechies et al. [8] observed that the concept of frame can be used to express functions in $L^2(\mathbb{R})$ in series expansions that differ from the orthonormal basis in the sense of flexibility and redundancy. Since the theory of frames and Gabor systems

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