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EXACT SOLUTIONS FOR WICK-TYPE STOCHASTIC SAWADA-KOTERA EQUATION

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Abstract. The variable coefficients Sawada-Kotera equations and Wick-type stochastic Sawada-Kotera equations are investigated. White noise functional solutions are shown by using the Hermite transform, homogeneous balance principle and the modified tanh-coth method. Moreover, some examples are given for the investigated model.

1. Introduction

It is well known that the solitons are stable against mutual collisions and behave like particles. In this sense, it is very important to study the nonlinear equations in random environment. However, variable coefficients nonlinear equations, as well as constant coefficients equations, cannot describe the realistic physical phenomena exactly. M. Wadati [17] first answered the interesting question. How does external noise affect the motion of solitons? and studied the diffusion of soliton of the KdV equation under Gaussian noise, which satisfies a diffusion equation in transformed coordinates. Wadati and Akutsu also studied the behaviors of solitons under the Gaussian white noise of the stochastic KdV equations with and without damping [19]. In addition, a nonlinear partial differential equation which describes wave propagations in random media was presented by Wadati [18]. Now, many researchers pay more attention to the study of the random waves, which are important subjects of stochastic partial differential equations, a number of soliton solutions of nonlinear stochastic partial differential equations and nonlinear stochastic fractional partial differential equations are obtained [1,2,3,4,5,6,7,8,9,10,11,12,13,14,29,30,31]. Holden et al.[11] researched stochastic partial

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differential equations in Wick versions via white noise functional approach. By using the homogeneous balance method, Xie studied exact solutions for stochastic mKdV equations [31]. Chen and Li used the symmetry reductions to study the soliton-like solutions for stochastic mKdV equation [2]. In this paper, with the help of Hermite transformation, white noise theory and modified tanh-coth method, we will deduce exact solutions for the Wick-type stochastic Sawada -Kotera equation as the following form:

$$P(t)\diamond U_t + Q(t)\diamond U_{5x} + R(t)\diamond (U\diamond U_{xxx} + U_x \diamond U_{xx}) + S(t)\diamond U^{\diamond 2}\diamond U_x = 0, \tag{1.1}$$

where " $\diamond$ " is the Wick product on the Kondratiev distribution space $(\mathcal{S})_{-1}$which was defined in [11],$P(t), Q(t), R(t)$and$S(t)$are$(\mathcal{S})_{-1}-$valued functions. Moreover, when Wick product " $\diamond$ "is replaced by ordinary product in Eq.(1.1), we obtain the Sawada -Kotera equation with variable coefficients:

$$p(t)u_t + q(t)u_{5x} + r(t)(uu_{xxx} + u_x u_{xx}) + s(t)u^2 u_x = 0, \tag{1.2}$$

which is of importance in mathematical physics field [15,16], where $p(t), q(t), r(t), s(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. And Eq. (1.1) can be regarded as the perturbation of Eq. (1.2).

2. White noise functional solutions of (1.1)

Taking the Hermite transform of Eq.(1.1), we get

$$\begin{aligned}\widetilde{P}(t,z)\widetilde{U}_t(x,t,z) + \widetilde{Q}(t,z)\widetilde{U}_{5x}(x,t,z) + \widetilde{R}(t,z)[\widetilde{U}(x,t,z)\widetilde{U}_{xxx}(x,t,z)\\ +\widetilde{U}_x(x,t,z)\widetilde{U}_{xx}(x,t,z)] + \widetilde{S}(t,z)\widetilde{U}^2(x,t,z)\widetilde{U}_x(x,t,z) = 0,\end{aligned} \tag{2.1}$$

where$z = (z_1, z_2, ...) \in \mathbb{C}^{\mathbb{N}}$is a vector parameter. Using$\widetilde{U}(x,t,z) = \widetilde{U}(\xi, z)$,$\xi = x - ct$that will carry out Eq. (2.1) into

$$-c\lambda_1 u_\xi + \lambda_2 u_{5\xi} + \lambda_3(uu_{\xi\xi\xi} + u_\xi u_{\xi\xi}) + \lambda_4 u^2 u_\xi = 0, \tag{2.2}$$

where $u(\xi, z) = \widetilde{U}(\xi, z)$,$\lambda_1(t,z) = \widetilde{P}(t,z)$,$\lambda_2(t,z) = \widetilde{Q}(t,z)$,$\lambda_3(t,z) = \widetilde{R}(t,z)and\lambda_4(t,z) = \widetilde{S}(t,z)$. Considering the homogeneous balance between $u_{5\xi}$and$u^2 u_\xi$ in Eq.(2.2), gives$M = 2$. According to the tanh-coth method [20-28], we can set the solution of Eq.(2.1) in the form

$$\begin{aligned}u(x,t,z) = S(Y) = a_0(t,z) + a_1(t,z)Y(\xi) + a_2(t,z)Y^2(\xi)\\ +b_1(t,z)Y^{-1}(\xi) + b_2(t,z)Y^{-2}(\xi),\end{aligned} \tag{2.3}$$

where $Y(\xi)$ satisfies the Riccati equation

$$Y' = c_1 + c_2 Y + c_3 Y^2, \tag{2.4}$$

and$c_1, c_2, c_3$are constant to be prescribed later. By virtue of (2.3) and (2.4) with observation of the linear independence of$Y^n (n = -7, -6, ..., 7)$Eq.(2.2) implies the following system of nonlinear equations

$$\begin{aligned}&-c\lambda_1 A_0 + \lambda_2 E_0 + \lambda_3[a_0 C_0 - a_1 C_6 - a_2 C_7 + b_1 C_1 + b_2 C_2] + A_0[\lambda_3 B_0 \lambda_4 F_0]\\ &- \quad A_4[\lambda_3 B_1 + \lambda_4 F_1] - A_5[\lambda_3 B_2 + \lambda_4 F_2] - A_6[\lambda_3 B_3 + \lambda_4 F_3]\\ &+ \quad A_1[\lambda_3 B_5 + \lambda_4 F_5] + A_2[\lambda_3 B_6 + \lambda_4 F_6] + A_3[\lambda_3 B_7 + \lambda_4 F_7] = 0,\end{aligned}$$

$$\begin{aligned}-c\lambda_1A_1+\lambda_2E_1+\lambda_3[a_0C_1+a_1C_0-a_2C_6+b_1C_2+b_2C_3]+A_1[\lambda_3B_0+\lambda_4F_0]\\ + \quad A_0[\lambda_3B_1+\lambda_4F_1]-A_4[\lambda_3B_2+\lambda_4F_2]-A_5[\lambda_3B_3+\lambda_4F_3]\\ - \quad A_6[\lambda_3B_4+\lambda_4F_4]+A_2[\lambda_3B_5+\lambda_4F_5]+A_3[\lambda_3B_6+\lambda_4F_6]=0,\end{aligned}$$

$$\begin{aligned}-c\lambda_1A_2+\lambda_2E_2+\lambda_3[a_0C_2+a_1C_1+a_2C_0+b_1C_3+b_2C_4]+A_2[\lambda_3B_0+\lambda_4F_0]\\ + \quad A_1[\lambda_3B_1+\lambda_4F_1]+A_0[\lambda_3B_2+\lambda_4F_2]-A_4[\lambda_3B_3+\lambda_4F_3]\\ - \quad A_5[\lambda_3B_4+\lambda_4F_4]+A_3[\lambda_3B_5+\lambda_4F_5]=0,\end{aligned}$$

$$\begin{aligned}-c\lambda_1A_3+\lambda_2E_3+\lambda_3[a_0C_3+a_1C_2+a_2C_1+b_1C_4+b_2C_5]+A_3[\lambda_3B_0+\lambda_4F_0]\\ + \quad A_2[\lambda_3B_1+\lambda_4F_1]+A_1[\lambda_3B_2+\lambda_4F_2]+A_0[\lambda_3B_3+\lambda_4F_3]-A_4[\lambda_3B_4+\lambda_4F_4]=0,\end{aligned}$$

$$\begin{aligned}\lambda_2E_4+\lambda_3[a_0C_4+a_1C_3+a_2C_2+b_1C_5]+A_3[\lambda_3B_1+\lambda_4F_1]\\ + \quad A_2[\lambda_3B_2+\lambda_4F_2]+A_1[\lambda_3B_3+\lambda_4F_3]+A_0[\lambda_3B_4+\lambda_4F_4]=0,\end{aligned}$$

$$\begin{aligned}\lambda_2E_5+\lambda_3[a_0C_5+a_1C_4+a_2C_3]+A_3[\lambda_3B_2+\lambda_4F_2]\\ + \quad A_2[\lambda_3B_3+\lambda_4F_3]+A_1[\lambda_3B_4+\lambda_4F_4]=0,\end{aligned}$$

$$\lambda_2E_6+\lambda_3[a_1C_5+a_2C_4]+A_3[\lambda_3B_3+\lambda_4F_3]+A_2[\lambda_3B_4+\lambda_4F_4]=0,$$

$$\lambda_2E_7+\lambda_3a_2C_5+A_3[\lambda_3B_4+\lambda_4F_4]=0,$$

$$\begin{aligned}c\lambda_1A_4-\lambda_2E_8+\lambda_3[-a_0C_6-a_1C_7-a_2C_8+b_1C_0+b_2C_1]-A_4[\lambda_3B_0+\lambda_4F_0]\\ - \quad A_5[\lambda_3B_1+\lambda_4F_1]-A_6[\lambda_3B_2+\lambda_4F_2]+A_0[\lambda_3B_5+\lambda_4F_5]+A_1[\lambda_3B_6+\lambda_4F_6]\\ + \quad A_2[\lambda_3B_7+\lambda_4F_7]+A_3[\lambda_3B_8+\lambda_4F_8]=0,\end{aligned}$$

$$\begin{aligned}c\lambda_1A_4-\lambda_2E_9+\lambda_3[-a_0C_7-a_1C_8-a_2C_9-b_1C_6+b_2C_0]-A_5[\lambda_3B_0+\lambda_4F_0]\\ - \quad A_6[\lambda_3B_1+\lambda_4F_1]-A_4[\lambda_3B_5+\lambda_4F_5]+A_0[\lambda_3B_6+\lambda_4F_6]\\ + \quad A_1[\lambda_3B_7+\lambda_4F_7]+A_2[\lambda_3B_8+\lambda_4F_8]=0,\end{aligned}$$

$$\begin{aligned}c\lambda_1A_6-\lambda_2E_{10}+\lambda_3[-a_0C_8-a_1C_9-a_2C_10-b_1C_7-b_2C_8]\\ - \quad A_6[\lambda_3B_0+\lambda_4F_0]-A_5[\lambda_3B_5+\lambda_4F_5]-A_4[\lambda_3B_6+\lambda_4F_6]\\ + \quad A_0[\lambda_3B_7+\lambda_4F_7]+A_1[\lambda_3B_8+\lambda_4F_8]=0,\end{aligned}$$

$$\begin{aligned}-\lambda_2E_{11}-\lambda_3[a_0C_9+a_1C_{10}+b_1C_8+b_2C_7]-A_6[\lambda_3B_5+\lambda_4F_5]\\ - \quad A_5[\lambda_3B_6+\lambda_4F_6]-A_4[\lambda_3B_7+\lambda_4F_7]+A_0[\lambda_3B_8+\lambda_4F_8]=0,\end{aligned}$$

$$\begin{aligned}\lambda_2E_{12}+\lambda_3[a_0C_{10}+b_1C_9+b_2C_8]+A_6[\lambda_3B_6+\lambda_4F_6]\\ + \quad A_5[\lambda_3B_7+\lambda_4F_7]+A_4[\lambda_3B_8+\lambda_4F_8]=0,\end{aligned}$$

$$\lambda_2E_{13}+\lambda_3[b_1C_{10}+b_2C_9]+A_6[\lambda_3B_7+\lambda_4F_7]+A_5[\lambda_3B_8+\lambda_4F_8]=0,$$

$$\lambda_2E_{14}+\lambda_3b_2C_{10}+A_6[\lambda_3B_8+\lambda_4F_8]=0, \tag{2.5}$$

where,$A_0=a_1c_1-b_1c_3,\ A_1=a_1c_2+2a_2c_1,\ A_2=a_1c_3+2a_2c_2,\ A_3=2a_2c_3$,$A_4=b_1c_2+2b_2c_3,\ A_5=b_1c_1+2b_2c_2,\ A_6=2b_2c_1,\ B_0=c_1A_1+c_3A_4,\ B_1=c_2A_1+2c_1A_2$, $B_2=c_3A_1+2c_2A_2+3c_1A_3,\ B_3=2c_3A_2+3c_2A_3, B_4=3c_3A_3,\ B_5=c_2A_4+2c_3A_5$, $B_6=c_1A_4+2c_2A_5+3c_3A_6,\ B_7=2c_1A_5+3c_2A_6,\ B_8=3c_1A_6,\ C_0=c_1B_1-c_3B_5$, $C_1=c_2B_1+2c_1B_2,\ C_2=c_3B_1+2c_2B_2+3c_1B_3,\ C_3=2c_3B_2+3c_2B_3+4c_1B_4$,

$C_4 = 3c_3B_3 + 4c_2B_4$, $C_5 = 4c_3B_4$, $C_6 = c_2B_5 + 2c_3B_6$, $C_7 = c_1B_5 + 2c_2B_6 + 3c_3B_7$, $C_8 = 2c_1B_6 + 3c_2B_7 + 4c_3B_8$, $C_9 = 3c_1B_7 + 4c_2B_8$, $C_{10} = 4c_1B_8$, $D_0 = c_1C_1 + c_3C_6$, $D_1 = c_2C_1 + 2c_1C_2$, $D_2 = c_3C_1 + 2c_2C_2 + 3c_1C_3$, $D_3 = 2c_3C_2 + 3c_2C_3 + 4c_1C_4$, $D_4 = 3c_3C_3 + 4c_2C_4 + 5c_1C_5$, $D_5 = 4c_3C_4 + 5c_2C_5$, $D_6 = 5c_3C_5$, $D_7 = c_2C_6 + 2c_3C_7$, $D_8 = c_1C_6 + 2c_2C_7 + 3c_3C_8$, $D_9 = 2c_1C_7 + 3c_2C_8 + 4c_3C_9$, $D_{10} = 3c_1C_8 + 4c_2C_9 + 5c_3C_{10}$, $D_{11} = 4c_1C_9 + 5c_2C_{10}$, $D_{12} = 5c_1C_{10}$, $E_0 = c_1D_1 - c_3D_7$, $E_1 = c_2D_1 + 2c_1D_2$, $E_2 = c_3D_1 + 2c_2D_2 + 3c_1D_3$, $E_3 = 2c_3D_2 + 3c_2D_3 + 4c_1D_4$, $E_4 = 3c_3D_3 + 4c_2D_4 + 5c_1D_5$, $E_5 = 4c_3D_4 + 5c_2D_5 + 6c_1D_6$, $E_6 = 5c_3D_5 + 6c_2D_6$, $E_7 = 6c_3D_6$, $E_8 = c_2D_7 + 2c_3D_8$, $E_9 = c_1D_7 + 2c_2D_8 + 3c_3D_9$, $E_{10} = 2c_1D_8 + 3c_2D_9 + 4c_3D_{10}$, $E_{11} = 3c_1D_9 + 4c_2D_{10} + 5c_3D_{11}$, $E_{12} = 4c_1D_{10} + 5c_2D_{11} + 6c_3D_{12}$, $E_{13} = 5c_1D_{11} + 6c_2D_{12}$, $E_{14} = 6c_3D_{12}$, $F_0 = a_0^2 + 2(a_1b_1 + a_2b_2)$, $F_1 = (a_0a_1 + a_2b_1)$, $F_2 = a_1^2 + 2a_0a_2$, $F_3 = 2a_1a_2$, $F_4 = a_2^2$, $F_5 = 2(a_0b_1 + a_{b_2})$, $F_6 = b_1^2 + 2a_0b_2$, $F_7 = 2b_1b_2$, and $F_8 = b_2^2$.

In the remaining part of this section we will discuss and solve our problem for some special cases for the Riccati equation as follows:

A. $c_1 = c_2 = 1, c_3 = 0$.

This choice for the constants implies that

$$Y_1(\xi) = exp(\xi) - 1. \tag{2.6}$$

By the aid of MATHEMATICA, the above system of equations (2.5) can be solved for the following cases:

Case I: $a_1 = a_2 = b_1 = 0$,$a_0 = \frac{\pm\sqrt{4\lambda_3^2+\lambda_4(c\lambda_1-16\lambda_2)}-2\lambda_3}{\lambda_4}$,$\lambda_1 \neq 0$,$\lambda_3 \neq 0and16\lambda_2 \leq c\lambda_1$, $b_2 = \frac{\pm 54\lambda_3\sqrt{4\lambda_3^2+\lambda_4(c\lambda_1-16\lambda_2)}-208\lambda_3^2-1710\lambda_2\lambda_4}{\pm 4\sqrt{4\lambda_3^2+\lambda_4(c\lambda_1-16\lambda_2)}}$.

Using Eq.(2.6) and Eq.(2.3), then Eq.(2.1) has the solution:

$$u_1 = \frac{\pm\sqrt{4\lambda_3^2+\lambda_4(c\lambda_1-16\lambda_2)}-2\lambda_3}{\lambda_4} + \frac{\pm 54\lambda_3\sqrt{4\lambda_3^2+\lambda_4(c\lambda_1-16\lambda_2)}-208\lambda_3^2-1710\lambda_2\lambda_4}{\pm 4\sqrt{4\lambda_3^2+\lambda_4(c\lambda_1-16\lambda_2)}(\exp(x-ct)-1)^2}. \tag{2.7}$$

Case II: $a_1 = a_2 = b_2 = 0, \lambda_1 \neq 0, \lambda_3 \neq 0, \lambda_2 \leq c\lambda_1, a_0 = \frac{\pm\sqrt{\lambda_3^2+4\lambda_4(c\lambda_1-\lambda_2)}-\lambda_3}{2\lambda_4}, b_1 = \frac{-54\lambda_2}{\lambda_3}$.

Using Eq.(2.6) and Eq.(2.3), then Eq.(2.1) has the solution:

$$u_2 = \frac{\pm\sqrt{\lambda_3^2+4\lambda_4(c\lambda_1-\lambda_2)}-\lambda_3}{2\lambda_4} + \frac{-54\lambda_2}{\lambda_3(\exp(x-ct)-1)}. \tag{2.8}$$

B. $c_1 = -c_3 = 0.5, c_2 = 0$.

This choice for the constants implies that

$$Y_2(\zeta) = coth(\xi) \pm csch(\xi), \tag{2.9}$$

or

$$Y_3(\zeta) = tanh(\xi) \pm isech(\xi). \tag{2.10}$$

By the aid of MATHEMATICA, the above system of equations (2.5) can be solved for the following cases:
Case III: $a_0 = a_1 = a_2 = b_1 = 0, \lambda_3 \neq 0, b_2 = \frac{35\lambda_2 - 2c\lambda_1}{5\lambda_3}$.

Using Eq.(2.6) and Eq.(2.3), then Eq.(2.1) has the solution:

$$u_3 = \frac{35\lambda_2 - 2c\lambda_1}{5\lambda_3(coth(x-ct) \pm csch(x-ct))^2}, \tag{2.11}$$

$$u_4 = \frac{35\lambda_2 - 2c\lambda_1}{5\lambda_3(tanh(x-ct) \pm i\, sech(x-ct))^2}. \tag{2.12}$$

Case IV: $a_0 = a_1 = b_1 = b_2 = 0, \lambda_3 \neq 0, a_2 = \frac{4c\lambda_1 - 31\lambda_2}{2\lambda_3}$.

Using Eq.(2.6) and Eq.(2.3), then Eq.(2.1) has the solution:

$$u_5 = \frac{4c\lambda_1 - 31\lambda_2}{2\lambda_3}(coth(x-ct) \pm csch(x-ct))^2, \tag{2.13}$$

$$u_6 = \frac{4c\lambda_1 - 31\lambda_2}{2\lambda_3}(tanh(x-ct) \pm i\, sech(x-ct))^2. \tag{2.14}$$

Lemma 2.1. [11] *Suppose*$u(x,t,z)$*is a solution (in the usual strong pointwise sense) of Eq. (2.1) for*(x,t)*in some bounded open set*$G \subset \mathbb{R} \times \mathbb{R}_+$*, and for all*$z \in K_m(n)$*, for some*$m < \infty, n > 0$*. Moreover, suppose that*$u(x,t,z)$*and all its partial derivatives, which are involved in Eq.(2.1), are (uniformly) bounded for*$(x,t,) \in G \times K_m(n)$*, continuous with respect to*$(x,t) \in G$*for all*$z \in K_m(n)$*and analytic with respect to*$z \in K_m(n)$*, for all*$(x,t) \in G$*. Then there exists*$U(x,t) \in (\mathcal{S})_{-1}$*such that* $u(x,t,z) = \widetilde{U}(x,t)(z)$*, for all* $(x,t,z) \in G \times K_m(n)$*and*$U(x,t)$*solves (in the strong sense) Eq. (1.1) in*$(\mathcal{S})_{-1}$.

From Lemma 2.1, we know that there exists$U(x,t) \in (\mathcal{S})_{-1}$such that$u(x,t,z) = \widetilde{U}(x,t)(z)$for all$(x,t,z) \in G \times K_m(n)$, where$U(x,t)$is the inverse Hermite transformation of$u(x,t,z)$. Consequently, $U(x,t)$ solves Eq.(1.1). Hence, for $P(t) \neq 0, R(t) \neq 0, S(t) > 0$ and $Q(t) \leq cP(t)\backslash 16$ the white noise functional solutions of Eq.(1.1) are as follows:

$$U_1(x,t) = \frac{\theta(t) - 2R(t)}{S(t)} + \frac{54R(t) \diamond \theta(t) - 208R^{\diamond 2}(t) - 171Q(t) \diamond S(t)}{4\theta(t) \diamond (exp^{\diamond}(x-ct) - 1)^{\diamond 2}}, \tag{2.15}$$

$$U_2(x,t) = \frac{\theta(t) - R(t)}{2S(t)} - \frac{54Q(t)}{R(t) \diamond (exp^{\diamond}(x-ct) - 1)}, \tag{2.16}$$

$$U_3(x,t) = \frac{35Q(t) - 2cP(t)}{5R(t) \diamond (coth^{\diamond}(x-ct) \pm csch^{\diamond}(x-ct))^{\diamond 2}}, \tag{2.17}$$

$$U_4(x,t) = \frac{35Q(t) - 2cP(t)}{5R(t) \diamond (tanh^{\diamond}(x-ct) \pm isech^{\diamond}(x-ct))^{\diamond 2}}, \tag{2.18}$$

$$U_5(x,t) = (4cP(t) - 31Q(t))(2R(t) \diamond (coth^{\diamond}(x-ct) \pm csch^{\diamond}(x-ct))^{\diamond 2}, \tag{2.19}$$

$$U_6(x,t) = (4cP(t) - 31Q(t))(2R(t) \diamond (tanh^{\diamond}(x-ct) \pm isech^{\diamond}(x-ct))^{\diamond 2}, \tag{2.20}$$

where

$$\theta(t) = \pm\{4R^{\diamond 2}(t) + S(t) \diamond [cP(t) - 16Q(t)]\}^{\diamond \frac{1}{2}}.$$

We observe that for different forms of$P(t), Q(t), R(t)$and$S(t)$, we can get different solutions of (1.1) from (2.15)-(2.20).

3. Examples

Let$W(t) = \dot{B}(t)$be the Gaussian white noise, where $B(t)$is the Brownian motion. We have the Hermite transform$\widetilde{W}(t,z) = \sum_{i=1}^{\infty} z_i \int_0^t \eta_i(s)ds$. Since$exp^{\diamond}[B(t)] = exp[B(t) - t^2/2]$, we have$tanh^{\diamond}[B(t)] = tanh[B(t) - t^2/2], coth^{\diamond}[B(t)] = coth[B(t) - t^2/2], sech^{\diamond}[B(t)] = sech[B(t) - t^2/2]andcsch^{\diamond}[B(t)] = csch[B(t) - t^2/2]$. Suppose $P(t) = Q(t) = \alpha R(t), R(t) = S(t) = f(t) + \beta W(t)$,where $\alpha, \beta$are arbitrary constants and $f(t)$is integrable or bounded measurable function on$\mathbb{R}_+$. Hence, the white noise functional solutions of Eq.(2.1) are as follows:

$$U_7(x,t) = \pm\sqrt{4 + \alpha(c - 16)} + \frac{R(t) \diamond \theta(t) - 208R^{\diamond 2}(t) - 1710\alpha S^{\diamond 2}(t)}{4\theta(t) \diamond [exp(x - c[t - \beta B(t) + \beta t^2 \setminus 2]) - 1]^2} - 2, \quad (3.1)$$

$$U_8(x,t) = \frac{1}{2}(\pm\sqrt{1 + 4\alpha(c - 16)} - 1) - \frac{54\alpha}{[exp(x - c[t - \beta B(t) + \beta t^2 \setminus 2]) - 1]}, \quad (3.2)$$

$$U_9(x,t) = \frac{\alpha(35 - 2c)}{[coth(x - c[t - \beta B(t) + \beta t^2 \setminus 2]) \pm csch(x - c[t - \beta B(t) + \beta t^2 \setminus 2])]^2}, \quad (3.3)$$

$$U_{10}(x,t) = \frac{\alpha(35 - 2c)}{[tanh(x - c[t - \beta B(t) + \beta t^2 \setminus 2]) \pm isech(x - c[t - \beta B(t) + \beta t^2 \setminus 2])]^2}, (3.4)$$

$$U_{11}(x,t) = \frac{\alpha}{2}(4c - 31)[coth(x - c[t - \beta B(t) + \beta t^2 \setminus 2]) \pm csch(x - c[t - \beta B(t) + \beta t^2 \setminus 2])]^2, (3.5)$$

$$U_{12}(x,t) = \frac{\alpha}{2}(4c - 31)[tanh(x - c[t - \beta B(t) + \beta t^2 \setminus 2]) \pm isech(x - c[t - \beta B(t) + \beta t^2 \setminus 2])]^2. (3.6)$$

4. Conclusion

In this paper, we have obtained many exact solutions to the Wick-type stochastic Sawada -Kotera equation via Hermite transformation and the modified tanh-coth method . These approaches can be used in various types of nonlinear stochastic equations, and help us to find more new stochastic excitations (especially the stochastic solitary wave excitations). In addition, we should also note that since there is a unitary map between the Wiener and the Poisson white noise spaces, we can obtain the solution of the Poissonian SPDE simply by applying this map to the solution of the corresponding Gaussian SPDE. A nice, concise account of this connection was given by Benth and Gjerde [1], and can be seen in Section 4.9 of [11] .Hence, we can get exact solutions of (1.1) if the coefficients $P(t), Q(t), R(t)$and$S(t)$are Poissonian white noise functionals.

References

[1] Benth. E and Gjerde. J, 1998, A Remark on the Equivalence between Poisson and Gaussian Stochastic Partial Differential Equations, Potential Anal., 8,179-193.

[2] Chen. Y and Li. B , 2005, The stochastic soliton-like solutions of stochastic mKdV equations, Czech. J. Phys. ,55,104-112.

[3] de Bouard .A and Debussche. A , 1998, On the Stochastic Korteweg-de Vries Equation, J. Funct. Anal., 154, 215-251.

[4] de Bouard .A and Debussche. A , 1999, White Noise Driven Korteweg-de Vries Equation , J. Funct. Anal., 169, 532-558.

[5] Debussche. A and Printems. J , 1999, Numerical simulation of the stochastic Korteweg-de Vries equation , Physica D : Nonlinear Phenomena, 134, 200-226.

[6] Debussche. A and Printems. J , 2001, Effect of a Localized Random Forcing Term on the Korteweg-de Vries Equation, Comput.Anal. Appl.,3,183-206.

[7] Ghany H A, 2011, Exact solutions for stochastic generalized Hirota-Satsuma Coupled KdV equations, Chin. J. Phys. 49 926

[8] Ghany H A and Hyder A, 2013, Local and global well-posedness of stochastic Zakharov-Kuznetsov equation., J. Comput. Anal. Appl. 15 1332

[9] Ghany H A and Saad M, 2012, White noise functional solutions for Wick-type stochastic fractional KdV-Burgers-Kuramoto equations, Chin. J. Phys. 50 618

[10] Ghany H A, Okb El Bab A, Zabel A and Hyder A, 2013, The fractional coupled KdV equations: Exact solutions and white noise functional approach, Chin. Phys. B 22 080501

[11] Holden. H, Øsendal. B, Ubøe. J and Zhang. T , Stochastic partial differential equations, Springer Science+Business Media, LLC, 2010.

[12] Konotop. VV and Vzquez. L, Nonlinear random waves, World Scientific, 1994.

[13] Okb El Bab A.S and H. A. Ghany 2010 Harmonic analysis on hypergroups, AIP Conf. Proc. 1309 312.

[14] Printems. J, 1999, The Stochastic Korteweg-de Vries Equation in$L_2(\mathbb{R})$, J. Differen. Equat. ,153,338-373.

[15] Satsuma. J and Kaup. D, 1977, A Bäcklund transformation for a higher order Korteweg-de Vries equation, J. Phys.Soc.Japan , 43,no. 2,692-697.

[16] Sawada. K and Kotera. T, 1974, A method for finding N-soliton solutions of the KdV equation and KdV like equation, Prog. Theor.Phys, 51, 1355-1367.

[17] Wadati. M, 1983, Stochastic Korteweg-de Vries Equation, J. Phys. Soc. Jpn. ,52,2642-2648.

[18] Wadati. M, 1990, Deformation of Solitons in Random Media, J. Phys. Soc. Jpn. ,59,4201-4203 .

[19] Wadati. M and Akutsu. Y, 1984, Stochastic Korteweg-de Vries Equation with and without Damping, J. Phys. Soc. Jpn. ,53,3342-3350.

[20] Wazwaz. A. M. , 2007, The extended tanh method for new solitons solutions for many forms of the fifth-order KdV equations, Applications and applied mathematics, 84, 1002-1014.

[21] Wazwaz. A. M., 2007, The extended tanh method for abundant solitary wave solutions of nonlinear wave equations, Applied Mathematics and Computation, 187, 1131-1142.

[22] Wazwaz. A. M., 2007, The tanh-coth and the sech methods for exact solutions of the Jaulent-Miodek equation, Physics Letters A, 366, 85-90.

[23] Wazwaz. A. M., 2008, The tanh-coth and the sine-cosine methods for kinks, solitons, and periodic solutions for the Pochhammer-Chree equations, Applied Mathematics and Computation, 195, 24-33.

[24] Wazwaz. A. M., 2008, The Hirota's bilinear method and the tanh-coth method for multiple-soliton solutions of the Sawada-Kotera-Kadomtsev-Petviashvili equation, Applied Mathematics and Computation, 200, 160-166.

[25] Wazwaz. A. M., 2008, The extended tanh method for new compact and noncompact solutions for the KP-BBM and the ZK-BBM equations, Chaos, Solitons & Fractals, 38, 1505-1516.
[26] Wazwaz. A. M., 2008, The tanh method for travelling wave solutions to the Zhiber-Shabat equation and other related equations, Communications in Nonlinear Science and Numerical Simulation, 13, 584-592.
[27] Wazwaz. A. M., 2008, The extended tanh method for the Zakharov-Kuznetsov (ZK) equation, the modified ZK equation, and its generalized forms, Communications in Nonlinear Science and Numerical Simulation, 13, 1039-1047.
[28] Wazzan. L, 2009, A modified tanh-coth method for solving the general Burgers-Fisher and the Kuramoto-Sivashinsky equations, Commun Nonlinear Sci Numer Simulat, 14, 2642-2652.
[29] Wei. CM, Ren. YH and Xia. ZQ , 2005, Exact soliton-like solutions for stochastic combined Burgers-KdV equation, Chaos, Solitons & Fractals ,26,329-336.
[30] Xie. Y.C, 2003, Exact solutions for stochastic KdV equations, Phys. Lett. A. , 310, 161-167.
[31] Xie. YC, 2004, Exact solutions for stochastic mKdV equations, Chaos, Solitons & Fractals, 19, 509-513.

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