

(Q^*, s) - CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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Abstract. In this paper, new generalizations of contra - continuity by using Q^* - closed sets called (Q^*, s) - continuity are presented. Characterizations and properties of (Q^*, s) - continuous functions are discussed in detail. Finally, we obtain many important results in topological spaces.

1. Introduction

Levine (1960) introduced the notion of generalized closed (briefly g - closed) sets in topological spaces and showed that compactness, countably compactness, para compactness and normality etc are all g -closed hereditary. The notion of b - open sets were introduced by Andrijevic [2]. Miguel caldas, Saeid Jafari and Raja M. Latif [16] introduced and studied (b, s) - continuous function in topological spaces. In 1982, Mashhour et al [15] introduced the notion of pre-open sets and the pre-closed sets were defined in [7]. The notion of Q^* - closed sets in a topological space was introduced by Murugalingam and Lalitha [17] in 2010. Initiation of contra-continuity was due to Dontchev [6]. In this paper, new generalizations of contra-continuity by using Q^* - closed sets called (Q^*, s) - continuity are presented. Characterizations and properties of (Q^*, s) - continuous functions are discussed in detail. Finally, we obtain many important results in topological spaces. Throughout this paper, (X, τ) and (Y, σ) (or X and Y) stand for topological spaces. For any subset A of X , the closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively.

2. Preliminaries

Before entering to our work we recall the following definitions and results which are used in this paper.

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Definition 2.1. The space X is called submaximal if every dense subset of X is open in X .

Definition 2.2. [17] Let (X, τ) be a topological space. Let $A \subset X$. A is said to be Q^* - closed if $\text{int}(A) = \emptyset$ and A is closed.

Definition 2.3. [17] The set of all Q^* - closed sets with X is a topology. It is denoted by τ_{Q^*} . For simplicity, write $\tau_{Q^*} = \sigma^*$.

Definition 2.4. A topological space X is said to be extremally disconnected if the closure of every open set of X is open in X .

Definition 2.5. [3] A space X is said to be s - Urysohn if for each pair of distinct points x and y in X , there exist $U \in SO(X, x)$ and $V \in SO(X, y)$ such that $cl(U) \cap cl(V) = \emptyset$.

Definition 2.6. [5] A function $f : X \rightarrow Y$ is said to be θ - irresolute if for each point $x \in X$ and each semi open set V of Y containing $f(x)$, there exists a semi open set U of X containing x such that $f(cl(U)) \subset cl(V)$.

Definition 2.7. [2] A subset A of a space X is said to be b - open if $A \subseteq (cl(\text{int}(A))) \cup (\text{int}(cl(A)))$.

Definition 2.8. [16] A function $f : X \rightarrow Y$ is said to be (b, s) - continuous function if for each point $x \in X$ and each semi open set V of Y with $f(x) \in V$, there exists a b - open set U of X containing x such that $f(U) \subset cl(V)$.

Definition 2.9. [13] A subset A of a topological space (X, τ) is called

- (1) a pre closed set if $cl[\text{int}(A)] \subseteq A$
- (2) a pre open set if $A \subseteq \text{int}[cl(A)]$.

Definition 2.10. [19] A function $f : X \rightarrow Y$ is said to be (θ, s) - continuous function if for each point $x \in X$ and each semi open set V of Y with $f(x) \in V$, there exists an open set U of X containing x such that $f(U) \subset cl(V)$.

Definition 2.11. [9] A function $f : X \rightarrow Y$ is said to be (gb, s) - continuous function if for each point $x \in X$ and each semi open set V of Y with $f(x) \in V$, there exists an gb - open set U of X containing x such that $f(U) \subset cl(V)$.

Definition 2.12. [21] A subset A of a topological space X is said to be Q^* - countably compact if and only if every countable $\tau - Q^*$ - open covering of X has a finite subcover.

Definition 2.13. [21] A topological space X is said to be Q^* - Lindelof if every Q^* - open cover of X has a countable sub cover.

Definition 2.14. [24] A function $f : X \rightarrow Y$ is said to be weakly b - continuous (briefly w.b.c) if for each point $x \in X$ and each open set V of Y with $f(x) \in V$, there exists an b - open set U of X containing x such that $f(U) \subset cl(V)$.

3. (Q^*, s) - continuous function

Definition 3.1. A function $f : X \rightarrow Y$ is said to be (Q^*, s) continuous function if for each point $x \in X$ and each semi open set V of Y with $f(x) \in V$, there exists a Q^* - open set U of X containing x such that $f(U) \subset cl(V)$.

Example 3.2. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a\}\}$. Let $f : X \rightarrow Y$ be the identity map. Hence the function f is (Q^*, s) - continuous function.

Example 3.3. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}\}$ and $Y = \{1, 2, 3\}$, $\sigma = \{\phi, Y, \{1\}\}$. Let $f : X \rightarrow Y$ be defined as $f(a) = f(c) = 1$ and $f(b) = 2$. Hence the function f is (Q^*, s) - continuous function.

Definition 3.4. A function $f : X \rightarrow Y$ is said to be (p, s) continuous function if for each point $x \in X$ and each semi open set V of Y with $f(x) \in V$, there exists a pre - open set U of X containing x such that $f(U) \subset cl(V)$.

Definition 3.5. A topological space X is said to be $Q^* - T_0$ if for each pair of distinct points x and y of X , there exist Q^* - open set A containing x but not y or Q^* - open set B containing y but not x .

Definition 3.6. A topological space X is said to be $Q^* - T_1$ if for each pair of distinct points x and y of X , there exists a Q^* - open sets A and B containing x and y , respectively such that $x \notin B$ and $y \notin A$.

Definition 3.7. A topological space X is said to be $Q^* - T_2$ for each pair of distinct points x and y in X , there exist disjoint Q^* - open sets A and B in X such that $x \in A$ and $y \in B$.

Theorem 3.8. If $f : X \rightarrow Y$ is a (Q^*, s) - continuous injection and Y is s - Urysohn , then X is $Q^* - T_2$.

Proof. Let x and y be distinct points in X . Then $f(x) \neq f(y)$ (since f is (Q^*, s) - continuous injection). To prove : X is $Q^* - T_2$. i.e., To prove : there exists $U \in Q^*O(X, x)$ and $V \in Q^*O(X, y)$ such that $U \cap V = \phi$. Since Y is s - Urysohn, there exist $V \in SO(Y, f(x))$ and $W \in SO(Y, f(y))$ such that $cl(V) \cap cl(W) = \phi$. Since f is (Q^*, s) - continuous, there exists $U \in Q^*O(X, x)$ and $G \in Q^*O(X, y)$ such that $f(U) \subset cl(V)$ and $f(G) \subset cl(W)$. $\implies f(U) \cap f(G) \subset cl(V) \cap cl(W) = \phi$. $\implies f(U \cap G) \subset \phi$. It follows that $U \cap G = \phi$. This shows that X is $Q^* - T_2$. $\square$

Theorem 3.9. Let $f, g : X \rightarrow Y$ be functions and $Q^* - cl(S)$ be Q^* - closed for each $S \subset X$. If

- (1) f and g are (Q^*, s) - continuous,
- (2) Y is s - Urysohn, then $A = \{x \in X : f(x) = g(x)\}$ is Q^* - closed in X .

Proof. Let $x \in X \setminus A$, then it follows that $f(x) \neq g(x)$. Since Y is s - Urysohn, there exist $M \in SO(Y, f(x))$ and $N \in SO(Y, g(x))$ such that $cl(M) \cap cl(N) = \phi$. Since f and g are (Q^*, s) - continuous, there exist Q^* - open sets U and V containing x such that $f(U) \subset cl(M)$ and $g(V) \subset cl(N)$. Hence, $U \cap V = P \in Q^*O(X)$, $f(P) \cap g(P) = \phi$ and then $x \notin Q^* - cl(A)$. Thus, A is Q^* - closed in X . $\square$

Theorem 3.10. If $f, g : X \rightarrow Y$ are (Q^*, s) - continuous functions , X is submaximal extremally disconnected and Y is s - Urysohn, then $E = \{x \in X : f(x) = g(x)\}$ is closed in X .

Proof. Suppose that $x \notin X$. Then $f(x) \neq g(x)$. Since Y is s - Urysohn, there exist $V \in SO(Y, f(x))$ and $W \in SO(Y, g(x))$ such that $cl(V) \cap cl(W) = \phi$. Since f and g are (Q^*, s) - continuous, there exist $U \in Q^*O(X, x)$ and $G \in Q^*O(X, y)$ such that

$f(U) \subset cl(V)$ and $g(G) \subset cl(W)$, $D = U \cap G$. Since X is submaximal extremally disconnected we have $U \in \tau$ and $G \in \tau$. $\implies U \cap G = \phi$. $\implies D \in \tau$ containing x . Therefore, $D \cap E = \phi$. $\implies D \not\subseteq E$. ie., $D \subset E^c$ and D is open in X . Therefore E is closed in X . $\square$

Theorem 3.11. Let $f, g : X \rightarrow Y$ be functions and $Q^* - cl(S)$ be Q^* - closed for each $S \subset X$. If

- (1) Y is s - Urysohn,
- (2) f and g are (Q^*, s) - continuous,
- (3) $f = g$ on Q^* - open set A subset X , then $f = g$ on X .

Proof. Since f and g are (Q^*, s) - continuous and Y is s - Urysohn, by Theorem 3.2, $B = \{x \in X : f(x) = g(x)\}$ is Q^* - closed in X . We have $f = g$ on Q^* - open set $A \subset X$. Since $A \subset B$ and A is Q^* - open set in X , then $X = cl(A) \subset cl(B) = B$. Hence $f = g$ on X . $\square$

Lemma 3.12. Let A be a subset of X and B be a subset of Y . If $A \in Q^*O(X)$ and $B \in Q^*O(Y)$, then $A \times B \in Q^*O(X \times Y)$.

Proof. Given that, A be a subset of X and B be a subset of Y . Let $A \in Q^*O(X)$ and $B \in Q^*O(Y)$. To prove : $A \times B \in Q^*O(X \times Y)$. Since $A \in Q^*O(X) \implies A \subseteq cl(A) = X$ and $B \subseteq Q^*O(Y) \implies B \subseteq cl(B) = Y$. It follows that $A \times B = cl(A) \times cl(B) = X \times Y$. Hence $A \times B \in Q^*O(X \times Y)$. $\square$

Theorem 3.13. If $f : X \rightarrow Y$ is a (Q^*, s) - continuous function and Y is s - Urysohn, then $E = \{(x, y) : f(x) = f(y)\}$ is Q^* - closed in $X \times X$.

Proof. Suppose that $(x, y) \notin E$. Then $f(x) \neq f(y)$. Since Y is s - Urysohn, there exists $V \in SO(Y, f(x))$ and $W \in SO(Y, f(y))$ such that $cl(V) \cap cl(W) = \phi$. Since f is (b, s) - continuous, there exists $U \in Q^*O(X, x)$ and $G \in Q^*O(X, y)$ such that $f(U) \subset cl(V)$ and $f(G) \subset cl(W)$. Set $D = U \times G$. By lemma 3.1, $(x, y) \in D \in G \in Q^*O(X \times X)$ and $D \cap E = \phi$. Since D is Q^* - open, therefore E is Q^* - closed in $X \times X$. $\square$

Theorem 3.14. If $f : X \rightarrow Y$ is Q^* - irresolute and $g : Y \rightarrow Z$ is (Q^*, s) - continuous, then $g \circ f : X \rightarrow Z$ is (Q^*, s) - continuous.

Proof. Let $f : X \rightarrow Y$ is Q^* - irresolute and $g : Y \rightarrow Z$ is (Q^*, s) - continuous. To prove : $g \circ f : X \rightarrow Z$ is (Q^*, s) - continuous. Let $x \in X$ and W be a semi open in Z containing $(g \circ f)(x)$. Since g is (Q^*, s) - continuous, there exists $V \in Q^*O(Y, f(x))$ such that $g(V) \subset cl(W)$. Since f is Q^* - irresolute, there exists $U \in Q^*O(X, x)$ such that $f(U) \subset V$. This shows that $(g \circ f)(U) = g(f(U)) \subset g(V) \subset cl(W)$. $\implies (g \circ f)(U) \subset cl(W)$. Therefore, $g \circ f$ is (Q^*, s) - continuous. $\square$

Theorem 3.15. If $f : X \rightarrow Y$ is (Q^*, s) - continuous and $g : Y \rightarrow Z$ is θ - irresolute, then $g \circ f : X \rightarrow Z$ is (Q^*, s) - continuous.

Proof. Given, $f : X \rightarrow Y$ is (Q^*, s) - continuous and $g : Y \rightarrow Z$ is θ - irresolute. To prove : $g \circ f : X \rightarrow Z$ is (Q^*, s) - continuous. Let $x \in X$ and W be a semi - open set in Z with $(g \circ f)(x) \in W$. Now we have to prove, there exists a Q^* - open set U of X containing x such that $(g \circ f)(U) \subset cl(W)$. Since $g : Y \rightarrow Z$ is θ - irresolute, there exists a semi open set V of Y containing $f(x)$ such that $g(cl(V)) \subset cl(W)$. Since f is (Q^*, s) -

continuous. there exists a Q^* - open set U of X containing x such that $f(U) \subset cl(V)$. It follows that $(g \circ f)(U) \subset cl(W)$. Therefore, $g \circ f$ is (Q^*, s) - continuous. $\square$

Definition 3.16. A function $f : X \rightarrow Y$ is said to be Q^* - open if $f(v) \in Q^*O(Y)$ for every $v \in Q^*O(X)$.

Definition 3.17. A function $f : X \rightarrow Y$ is said to be Q^* - closed if $f(v) \in Q^*C(Y)$ for every $v \in Q^*C(X)$.

Theorem 3.18. If $f : X \rightarrow Y$ is a Q^* - open surjective function and $g : Y \rightarrow Z$ is a function such that $g \circ f$ is (Q^*, s) - continuous, then g is (Q^*, s) - continuous.

Proof. Given $f : X \rightarrow Y$ is a Q^* - open surjective function and $g : Y \rightarrow Z$ is a function such that $g \circ f : X \rightarrow Z$ is (Q^*, s) - continuous. To prove, g is (Q^*, s) - continuous. Suppose that x and y are in X and Y respectively, such that $f(x) = y$. Let W be a semi open set in Z containing $(g \circ f)(x) = g(f(x)) = g(y)$. Then there exists $U \in Q^*O(X, x)$ such that $(g \circ f)(U) = g(f(U)) \subset cl(W)$. Since f is Q^* - open. Then $f(U) \in Q^*O(Y, y)$. Thus, for each point $y \in Y$ and W be a semi open set in Z containing $g(y)$. There exist a Q^* - open set $f(U)$ in Y containing y such that $g(f(U)) \subset cl(W)$. This implies that g is (Q^*, s) - continuous. $\square$

Definition 3.19. Two subsets A and B of a space X are called Q^* - separated if there exists a Q^* - open sets U and V such that $A \subset U$ and $B \subset V$.

Definition 3.20. A Space X is called Q^* - normal if any pair of disjoint Q^* - closed subsets A and B of X , there exist disjoint Q^* - open subsets U and V such that $A \subset U$ and $B \subset V$.

Theorem 3.21. If $f : X \rightarrow Y$ is (Q^*, s) - continuous, Q^* - closed function of a Q^* - normal space X onto a space Y , then any two disjoint θ - semi closed subsets of Y can be Q^* - separated.

Proof. Let F_1 and F_2 be any two disjoint θ - semi closed sets of Y . Since f is (Q^*, s) - continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint Q^* - closed sets of X . By Q^* - normality of X , there exists $U_1, U_2 \in Q^*O(X)$ such that $f^{-1}(F_1) \subset U_1, f^{-1}(F_2) \subset U_2$ and $U_1 \cap U_2 = \phi$. Let $V_i = Y - f(X - U_i)$ for $i = 1, 2$. Since f is Q^* - closed, the sets V_1, V_2 are Q^* - open in Y and $F_i \subset V_i$ for $i = 1, 2$. Since U_1 and U_2 are disjoint and $f^{-1}(F_i) \subset U_i$ for $i = 1, 2$. We obtain $V_1 \cap V_2 = \phi$. This shows that F_1 and F_2 are Q^* - separated. $\square$

Theorem 3.22. If $f : X \rightarrow Y$ is Q^* - irresolute and Q^* - closed surjection and X is a Q^* - normal, then Y is Q^* - normal.

Proof. Let F_1 and F_2 be any two disjoint Q^* - closed sets of Y . Since f is Q^* - irresolute, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint Q^* - closed sets of X . By Q^* - normality of X , there exists $U_1, U_2 \in Q^*O(X)$ such that $f^{-1}(F_1) \subset U_1, f^{-1}(F_2) \subset U_2$ and $U_1 \cap U_2 = \phi$. Let $V_i = Y - f(X - U_i)$ for $i = 1, 2$. Since f is Q^* - closed, the sets V_1, V_2 are Q^* - open in Y and $F_i \subset V_i$ for $i = 1, 2$. Since U_1 and U_2 are disjoint and $f^{-1}(F_i) \subset U_i$ for $i = 1, 2$. We obtain $V_1 \cap V_2 = \phi$. This shows that Y is Q^* - normal. $\square$

Definition 3.23. A space X is said to be semi compact [5] (resp. semi - countably compact) if every cover (resp. countable cover) of X by semi open sets has a finite subcover.

Theorem 3.24. Let $f : X \rightarrow Y$ be a (Q^*, s) - continuous surjection . Then the following statements hold:

- (1) If X is Q^* - Lindelof, then Y is semi - Lindelof.
- (2) If X is countably Q^* - compact, then Y is semi - countably compact.

Definition 3.25. A space X is called Q^* - connected if X is not the union of two disjoint nonempty Q^* - open sets.

Theorem 3.26. Let $f : X \rightarrow Y$ be a (Q^*, s) - continuous surjection. If X is Q^* - connected, then Y is connected.

Proof. Assume that Y is not connected space. Then there exists a nonempty disjoint open sets A and B such that $Y = A \cup B$. Also A and B are clopen in Y . Since f is (Q^*, s) - continuous , $f^{-1}(A)$ and $f^{-1}(B)$ are Q^* - open in X . Moreover $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty disjoint and $X = f^{-1}(A) \cup f^{-1}(B)$. This shows that X is not Q^* - connected. This contradicts the assumption that Y is not connected. $\square$

Theorem 3.27. Every (Q^*, s) - continuous function is (b, s) - continuous.

Proof. Let $f : X \rightarrow Y$ be a (Q^*, s) - continuous function. To prove that f is (b, s) - continuous. Let $x \in X$ and each semi open set V of Y with $f(x) \in V$. Since f is (Q^*, s) - continuous, there exists a Q^* - open set U of X containing x such that $f(U) \subset cl(V)$. Since every Q^* - open set is b - open we have f is (b, s) - continuous. $\square$

Remark 3.28. Converse of the theorem need not be true in general. The following example supports our claim.

Example 3.29. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ and $Y = \{1, 2, 3\}$, $\sigma = \{\phi, Y, \{2\}, \{3\}, \{2, 3\}, \{1, 3\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. Hence f is (b, s) continuous, but not (Q^*, s) continuous.

Theorem 3.30. Every (Q^*, s) - continuous function is (p, s) - continuous.

Remark 3.31. Converse of the theorem need not be true in general. The following example supports our claim.

Example 3.32. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}\}$ and $Y = \{1, 2, 3\}$, $\sigma = \{\phi, Y, \{1\}, \{2, 3\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. Hence f is s , (p, s) - continuous, but not (Q^*, s) - continuous.

Theorem 3.33. Every (θ, s) - continuous function is (b, s) - continuous.

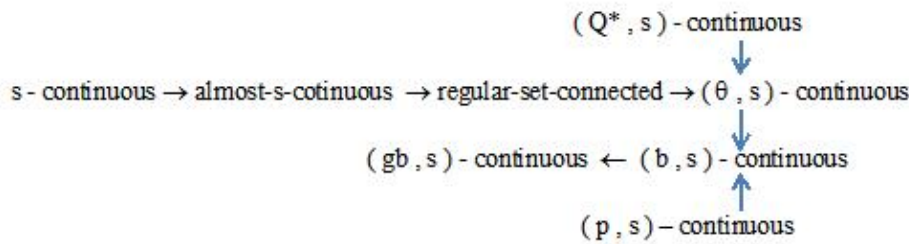
Remark 3.34. Converse of the theorem need not be true in general. The following example supports our claim.

Example 3.35. In example 3.4, f is (b, s) - continuous but not (θ, s) - continuous.

Remark 3.36. (Q^*, s) - continuous and Q^* - continuous maps are independent of each other in general. It is proved in the following example.

Example 3.37. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and $Y = \{1, 2, 3\}$, $\sigma = \{\phi, Y, \{2\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = 2$, $f(b) = 1$, $f(c) = 3$. Hence f is (Q^*, s) - continuous, but not Q^* - continuous.

Remark 3.38. From the above, we have the following diagram of implications.



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