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SOME REMARKS ON ADOMIAN DECOMPOSITION METHOD

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Abstract. Adomian Decomposition Method is a powerful method for solving general functional equations in Banach spaces. It provides a sequence of analytic approximate solutions to a wide range of nonlinear equations.

In this work we state the conditions under which, in the autonomous case, the approximate solutions to ordinary differential equations provided by Adomian Decomposition Method are the Taylor polynomials of the solution. We also offer an example where the approximations are not polynomials and highlight some limitations of the method.

1. Introduction

Adomian Decomposition Method (ADM), introduced by George Adomian in the '80s ([2], [3], [4]), is a powerful tool to solve a wide range of nonlinear equations. It produces analytical approximate solutions for very general nonlinear problems without linearization or simplification. The description of the method is simple, but it is difficult to give a general proof of convergence because of the wide range of problems it covers. The author gave no formal proof of the convergence but gave a justification based on the composition of Taylor series and offered lots of examples of application to problems with well known solutions, to show the goodness of the method. Since Adomian's first publications, a great number of works have appeared where ADM is proposed to solve functional, differential, integro-differential and algebraic equations, in a wide range of practical problems (see, for example, [7], [13], [16] and [20] as a small sample of them); also, many articles have been published comparing this method with other ones (we quote [12], [13] or [17], to give some few examples). Although it is theoretically possible to find approximate solutions to different type of equations by ADM, there exist some limitations and disadvantages that are not always mentioned when the method is applied. The **limitations** are mainly related to the strong hypothesis needed to assure

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the existence and convergence of the approximate solutions in a general case. The **disadvantages** are concerned principally with the accuracy of the solution and its local character.
In this work, we theoretically compare the approximations derived from the Taylor expansion of the solution to an ordinary differential equation with the one provided by ADM, and state the conditions under which, in the autonomous case, they coincide.
In what follows, we briefly describe the method. The comparison between the solution provided by ADM and the one obtained by Taylor expansion is included in Section 2. Some of the advantages, disadvantages and limitations of the method are also presented there, and we illustrate them with an example. Finally, we draw some conclusions.

1.1. The Adomian Decomposition Method. In order to settle the notation, we briefly describe the well-known Adomian Decomposition Method. We refer to [3] and [4] for a detailed description.
Let us consider a general functional equation of the form:

$$F[u(x)] = g(x) \tag{1.1}$$

where $F : X \to Y$ is an operator between two Banach (functional) spaces X and Y, conveniently chosen. The operator F is supposed to be decomposed as

$$F = L + R + N \tag{1.2}$$

where L is a linear and *easily* invertible operator, R is a linear operator and N is a non linear one. Thus, from (1.1) and (1.2), we have

$$u = L^{-1}[g(x)] - L^{-1}[R(u)] - L^{-1}[N(u)]. \tag{1.3}$$

In addition, it is assumed that N can be written in the form

$$N[u(x)] = \sum_{n=0}^{+\infty} A_n[u(x)],$$

where the A_n's are known as *Adomian's polynomials*, although they are not always polynomials in the usual sense. Under these hypothesis, an analytic solution u is supposed to exist and it is written as

$$u(x) = \sum_{n=0}^{+\infty} u_n(x). \tag{1.4}$$

The goal of the method is that each polynomial A_n can be build depending only on $u_0, u_1, \cdots u_n$ thus, from (1.3), it is possible to find a recursive formula for the terms in (1.4):

$$\begin{cases} u_0(x) = & L^{-1}[g(x)] \\ u_n(x) = & -L^{-1}[R(u_{n-1}(x)] - L^{-1}[A_{n-1}(u_0(x), \cdots, u_{n-1}(x)] \qquad \forall n \geq 1, \end{cases} \tag{1.5}$$

Initial or boundary conditions are taken into account when $u_0(x)$ is obtained.
In [4] G. Adomian stated that partial sums

$$\varphi_m = \sum_{n=0}^{m} u_n \tag{1.6}$$

are good approximations of the solution, and converge rapidly to it. He also remarked that this method does not provide a numerical solution, but an approximate analytical

one; consequently, if the approximation is *good* enough, properties of the solution that could be theoretically derived, could be observed.
The description of the method is simple, but the hypothesis on the operator F (that is, on L and N) are very restrictive and not easy to be verified. Nor is it easy to give a general proof of convergence.
Regarding the construction of the Adomian's polynomials A_n, Adomian himself and other authors described different ways to build them up. We refer to [2], [3] and [4] and the improvements proposed in [5], [8], [21] and [24]. In [14] and [15], supposing that N is *an analytical operator in a proper Banach space*, the author considers an auxiliary parameter ε, stating that

$$N(u,\varepsilon) = N(\sum_{n=0}^{+\infty} \varepsilon^n u_n) = \sum_{n=0}^{+\infty} A_n(u)\varepsilon^n,$$

from which

$$A_n(u) = \frac{1}{n!}\frac{d^n}{d\varepsilon^n}[N(\sum_{k=0}^{+\infty}\varepsilon^k u_k)]|_{\varepsilon=0} = \frac{1}{n!}\frac{d^n}{d\varepsilon^n}[N(\sum_{k=0}^{n}\varepsilon^k u_k)]|_{\varepsilon=0} = A_n(u_0,\cdots,u_n) \quad (1.7)$$

Several proofs of the convergence of the partial sums φ_m to the solution have been presented (see, for example, [1], [10], [11], [14], [15], [18], [21] and [23]) - although some of them are incomplete and imprecise, and the existence of an analytical solution is generally supposed but not always justified. In [1], a nice proof of the convergence of the method, in the case of a differential equation, is presented. Under the assumption of analyticity of N and recalling the Cauchy-Kowalevskaya Theorem, the authors justify the existence of an analytic solution and use majorants and Cauchy estimates to prove the convergence. In their demonstration it is clear that we can only expect small regions of convergence, and a process of concatenation might be necessary to build up a global solution (see also [23], and [6], [12] and [16] for examples of locality and concatenation).
Assuming the convergence of the series involved, the approximate solution (1.6) provided by ADM looks like the Taylor expansions of the solution u. It must be said that, although the terms u_n could be obtained by composition of Taylor series, *they are not always polynomials.* They depend on the choice of the linear operator to be inverted, L, the data function g and the initial or boundary conditions. In the next section we explore this issue.

2. Solution to ODEs by ADM

2.1. ADM and Taylor expansions. In the literature, there are many examples of approximate solutions, obtained by ADM, to ordinary autonomous initial value problems like

$$\begin{cases} u'(x) = & f(u(x)) \\ u(0) = & u_0 \end{cases} \quad (2.8)$$

with $f : U \subset \mathbb{R} \to \mathbb{R}$ and $U \subset \mathbb{R}$ an open set (see, for example, [6] and [13]).
The functional F and the function g in (1.1) are related to (2.8): $F(.) = \frac{d(.)}{dx} - f(.)$ and $g(x) \equiv 0$. It is enough to suppose that f is analytic to assure the existence of a unique analytic solution u.
In order to apply ADM, F is decomposed as $F = L + N + R$ and in most of the

applications L is chosen to be just the first derivative, $L(\cdot) = \frac{d(\cdot)}{dx}$. In this case, as $u_0 \in \mathbb{R}$ is constant, we will prove that the approximate solutions provided by ADM are exactly partial sums of Taylor series of the solution u.

Proposition 2.1. If we choose L in (1.2) as the first derivative with respect to x, then the approximate solution φ_m for the ordinary initial value problem (2.8) with f analytic, is the Taylor polynomial of order m of the solution u.

Proof:

The unique analytic solution to (2.8) can be written as $u(x) = \sum\limits_{n=0}^{+\infty} u_n(x)$ and considering $L(\cdot) = \frac{d(\cdot)}{dx}$, $R = 0$ and $N = -f$, by (1.5) the recursive formula leads to

$$\begin{cases} u_0(x) = & u_0 \\ u_n(x) = & \int\limits_0^x [A_{n-1}(u_0(s), \cdots, u_{n-1}(s))]ds, \qquad n \geq 1 \end{cases}$$

with

$$A_n(u_0, \cdots, u_n) = \frac{1}{n!}\frac{d^n}{d\varepsilon^n}[f(\sum_{k=0}^{n} \varepsilon^k u_k)]|_{\varepsilon=0}. \tag{2.9}$$

We recall a formula published in 1855 by Faà di Bruno (see [19]) for the successive derivatives of a composition:

$$\frac{d^n(f \circ u)}{dx^n}(0) = \sum_{j=1}^{n} \sum_{i_1, i_2, \ldots, i_n} \frac{n!}{i_1! i_2! \cdots i_n!} f^{(j)}(u(0))(\frac{u'(0)}{1!})^{i_1}(\frac{u''(0)}{2!})^{i_2} \cdots (\frac{u^{(n)}(0)}{n!})^{i_n} \tag{2.10}$$

where the interior sum is made over every solution of $i_1 + 2i_2 + \cdots + ni_n = n$ and $i_1 + i_2 + \cdots + i_n = j$, with $i_1, i_2, \ldots i_n \in \mathbb{N}_0$.
By induction we will prove that

$$u_n(x) = u^{(n)}(0)\frac{x^n}{n!}, \forall n \geq 0 \tag{2.11}$$

and, consequently, each partial sum $\varphi_m = \sum\limits_{n=0}^{m} u_n$ of the approximate solution provided by ADM is the Taylor polynomial of order m around $x = 0$ of the analytical solution u, i.e., ADM and the Taylor expansion coincides.
Since $u_0(x) = u_0 = u(0) = u^{(0)}(0)\frac{x^0}{0!}$, (2.11) is true for $n = 0$.
Let us suppose now that equation (2.11) is satisfied for all $j \leq n$.
By the recursive formula and using (2.9) we obtain,

$$u_{n+1}(x) = \int\limits_0^x [A_n(u_0(s), \cdots, u_{n-1}(s))]ds = \int\limits_0^x \frac{1}{n!}\frac{d^n}{d\varepsilon^n}[f(\sum_{k=0}^{n} \varepsilon^k u_k(s))]|_{\varepsilon=0} ds \tag{2.12}$$

Note that, if $g(s, \varepsilon) = \sum\limits_k \varepsilon^k u_k(s)$ and the series is convergent, then

$$u_k(s) = \frac{1}{k!}\frac{\partial^{(k)} g}{\partial \varepsilon^k}(s, 0). \tag{2.13}$$

Applying (2.10) to (2.12), using (2.13) and the inductive hypothesis, it results

$$\begin{aligned} u_{n+1}(x) = & \int_0^x \sum_{j=1}^n \sum_{i_1,i_2,\ldots,i_n} \tfrac{f^{(j)}(u(0))}{i_1!i_2!\cdots i_n!}(\tfrac{1}{1!}\tfrac{\partial g}{\partial \varepsilon}(s,0))^{i_1}(\tfrac{1}{2!}\tfrac{\partial^2 g}{\partial \varepsilon^2}(s,0))^{i_2}\cdots(\tfrac{1}{n!}\tfrac{\partial^{(n)} g}{\partial \varepsilon^n}(s,0))^{i_n} ds = \\ = & \int_0^x \sum_{j=1}^n \sum_{i_1,i_2,\ldots,i_n} \tfrac{f^{(j)}(u(0))}{i_1!i_2!\cdots i_n!}(u_1(s))^{i_1}(u_2(s))^{i_2}\cdots(u_n(s))^{i_n} ds = \\ = & \int_0^x \sum_{j=1}^n \sum_{i_1,i_2,\ldots,i_n} \tfrac{f^{(j)}(u(0))}{i_1!i_2!\cdots i_n!}(u'(0)\tfrac{s^1}{1!})^{i_1}(u''(0)\tfrac{s^2}{2!})^{i_2}\cdots(u^{(n)}(0)\tfrac{s^n}{n!})^{i_n} ds = \\ = & \sum_{j=1}^n \sum_{i_1,i_2,\ldots,i_n} \tfrac{f^{(j)}(u(0))}{i_1!i_2!\cdots i_n!}(\tfrac{u'(0)}{1!})^{i_1}(\tfrac{u''(0)}{2!})^{i_2}\cdots(\tfrac{u^{(n)}(0)}{n!})^{i_n}\int_0^x [s^{i_1+2i_2+\ldots+ni_n}]ds \end{aligned}$$

Recalling $i_1 + 2i_2 + \cdots + ni_n = n$, integrating and using (2.10) we obtain

$$\begin{aligned} u_{n+1}(x) = & \sum_{j=1}^n \sum_{i_1,i_2,\ldots,i_n} \tfrac{1}{i_1!i_2!\cdots i_n!} f^{(j)}(u(0))(\tfrac{u'(0)}{1!})^{i_1}(\tfrac{u''(0)}{2!})^{i_2}\cdots(\tfrac{u^{(n)}(0)}{n!})^{i_n}\tfrac{x^{n+1}}{n+1} = \\ = & \tfrac{1}{n!}\tfrac{d^n(f\circ u)}{dx^n}(0)\tfrac{x^{n+1}}{n+1} = \tfrac{d^n(f\circ u)}{dx^n}(0)\tfrac{x^{n+1}}{(n+1)!}. \end{aligned}$$

In addition, since $u'(x) = f(u)$ and $u(x)$ is analytic, we have $u^{(n+1)}(0) = \frac{d^n(f\circ u)}{dx^n}(0)$. Then, $u_{n+1}(x) = u^{(n+1)}(0)\frac{x^{(n+1)}}{(n+1)!}$ and the result follows.¶

For example, the approximate solution φ_4 to (2.8) is

$$\begin{aligned} u(x) \simeq \varphi_4(x) = & \; u_0 + f(u_0)x + f'(u_0)f(u_0)\tfrac{x^2}{2} + [f''(u_0)f^2(u_0) + (f'(u_0))^2 f(u_0)]\tfrac{x^3}{3!} + \\ + & \; [f'''(u_0)f^3(u_0) + 4f''(u_0)f'(u_0)f^2(u_0) + f(u_0)(f'(u_0))^3]\tfrac{x^4}{4!} \end{aligned}$$

This expression could be obtained in a simpler way, by the traditional procedure of deriving successively the given differential equation. Although in this case ADM is nothing but a more difficult method to obtain Taylor series, several of the examples offered in the literature correspond to this situation.

Note that a similar result might be proved for a *system* of autonomous ODEs. In [9], [16], [20] and [22], examples of this type are presented. In all of them, the Taylor polynomial is obtained by means of ADM.

2.2. Different approximate solutions by ADM. If L in (1.2) is not chosen to be the first derivative, ADM applied to (2.8) may not lead to a Taylor series. In this case the approximate solutions provided by ADM are, in general, not polynomials. The following example illustrates this situation.

Let us consider the problem

$$\begin{cases} u'(x) = & u(1-ku) \\ u(0) = & a \end{cases} \tag{2.14}$$

for which the exact solution is $u(x) = ae^x[1 - ak(1 - e^x)]^{-1}$.

To build an approximate solution, instead of choosing L to be just the first derivative, it is possible to select $L(u) = u' - u$ and consequently $L^{-1}(.) = \int e^{x-s}(.)ds$.

For this choice of L, $R \equiv 0$, $g \equiv 0$ and $N(u) = ku^2$, the recursive formula results

$$\begin{cases} u_0(x) = & L^{-1}(0) \\ u_n(x) = & -L^{-1}[A_{n-1}(u_0(x), \cdots, u_{n-1}(x)] \qquad \forall n \geq 1, \end{cases}$$

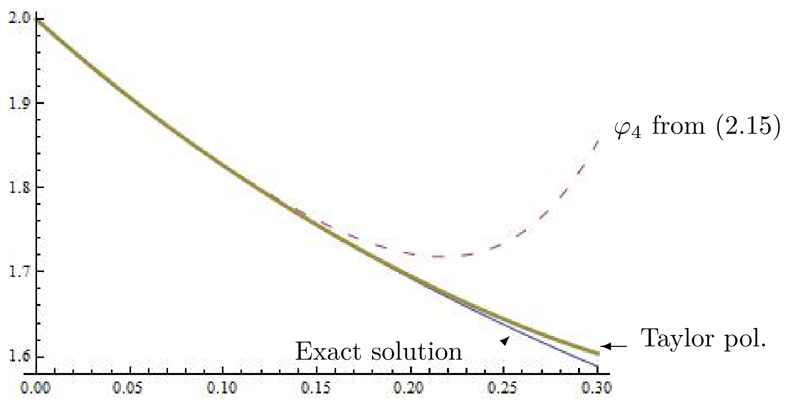

FIGURE 1. The exact solution of (2.14) and two ADM approximations.

where $A_n(u_0(x), \cdots, u_n(x))$ is the Adomian polynomial of order n for $N(u)$.
The approximate solution is now build up:

$$\begin{aligned}
u_0(x) = &\quad ae^x \\
A_0(u_0) = &\quad ka^2e^{2x} \\
u_1(x) = &\quad -e^x \int_0^x e^{-s}[A_0(u_0(s)]ds = ae^x ak(1-e^x) \\
A_1(u_0,u_1) = &\quad \tfrac{d}{d\varepsilon}[(u_0+\varepsilon u_1)^2]|_{\varepsilon=0} = 2kae^x u_1 \\
u_2(x) = &\quad -e^x \int_0^x e^{-s}[A_1(u_0(s),u_1(s)]ds = ae^x a^2k^2(1-e^x)^2 \\
A_2(u_0,u_1,u_2) = &\quad \tfrac{1}{2!}\tfrac{d^2}{d\varepsilon^2}[(u_0+\varepsilon u_1+\varepsilon^2 u_2)^2]|_{\varepsilon=0} = ku_1^2 + 2kae^x u_2 \\
u_3(x) = &\quad -e^x \int_0^x e^{-s}[A_2(u_0(s),u_1(s),u_2(s))]ds = ae^x a^3k^3(1-e^x)^3 \\
A_3(u_0,u_1,u_2,u_3) = &\quad \tfrac{1}{3!}\tfrac{d^3}{d\varepsilon^3}[(u_0+\varepsilon u_1+\varepsilon^2 u_2+\varepsilon^3 u_3)^2]|_{\varepsilon=0} = 2ku_1u_2 + 2kae^x u_3 \\
u_4(x) = &\quad -e^x \int_0^x e^{-s}[A_3(u_0(s),u_1(s),u_2(s),u_3(s))]ds = ae^x a^4k^4(1-e^x)^4
\end{aligned}$$

etc.. The approximate solution looks like

$$\begin{aligned}
u(x) \simeq &\quad ae^x[1+ak(1-e^x)+(ak(1-e^x))^2+(ak(1-e^x))^3+\ldots\ ] = \\
= &\quad ae^x \textstyle\sum_{n=0}^{\infty}(ak(1-e^x))^n
\end{aligned} \tag{2.15}$$

which obviously leads to $u(x) = ae^x[1-ak(1-e^x)]^{-1}$ in $D = \{x \in \mathbb{R} : |ak(1-e^x)| \le 1\}$. In Fig. 1 the exact solution, its Taylor polynomial of fourth degree and a partial sum of (2.15) up to u_4 are shown, for $a = 2, k = 1$. It can be noticed that the Adomian approximation obtained using the non trivial inverse operator $L^{-1}(.) = \int e^{x-s}(.)ds$ takes away from the exact solution more rapidly than the Taylor polynomial - which would be obtained considering $L(.) = \frac{d(.)}{dx}$, so it is plausible to wonder about the real advantages of using ADM in this case. Besides that, whichever approximate solution is chosen, it fits to the exact solution within a tolerance of 1% only for a small domain: 0.3 for Taylor series and 0.18 for φ_4. To build up an analytical approximation in a bigger domain, a concatenation process is needed, which usually demands considerable long computational time, and a careful control of the propagation of errors (see [13] and [22], for example). These considerations are not generally mentioned in the publications.

However, it is worth noting that, for this example, choosing $L^{-1}(.) = \int e^{x-s}(.)ds$, ADM leads to a closed formula for the solution in a neighborhood around 0.

3. Conclusions

Adomian's Decomposition Method is a tool introduced in the '80s for solving general nonlinear equations. It provides an analytical approximate solution that, in some cases, is simply the Taylor series, but in others could be a non trivial convergent series depending on the linear operator L chosen to be inverted. In the case of autonomous ODEs, if L is chosen as the first derivative, we have proved that ADM provides simply a Taylor polynomial expansion of the solution, which could be obviously obtained directly, by the traditional procedure of deriving successively the given differential equation. Since the approximate solution provided by this last method is, in general, easier to build, in those cases the ADM offers no worthwhile contribution. When the operator L is chosen not to be simply the derivative, the solution obtained by means of ADM is not the Taylor expansion, it may be not even a polynomial. In these cases it is worth to notice the extremely local character of the solution and the usually long computational time needed to build it up. These considerations suggest that the method, in such cases, is more interesting from a theoretical point of view than from a practical one.

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