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PICTURE FUZZY MINIMAL STRUCTURE SPACES AND ITS APPLICATIONS

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Abstract. This paper is an introduction of picture fuzzy minimal structure space and discusses properties of picture fuzzy minimal structure space. It has plenty of applications. This motivates us to present the concept of picture fuzzy minimal structure space. We define picture fuzzy minimal structure space, closure and interior of a set and subspace of picture fuzzy minimal structure space. Finally, as an application, a decision making problem is solved using picture fuzzy minimal structure with a score function.

1. Introduction

Zadeh's [21] Fuzzy set laid the foundation of many theories such as intuitionistic fuzzy set and picture fuzzy set, rough sets etc. Later, researchers developed K. T. Atanassov's [1] intuitionistic fuzzy set theory in many fields such as differential equations, topology, computer science and so on. F. Smarandache [19, 20] found that some objects have indeterminacy or neutral other than membership and non-membership. So, he coined the notion of neutrosophy. In this era, Researchers [8, 9, 12, 13, 14] applied the concept of neutrosophy when object has inconsistent, incomplete information. Picture fuzzy set theory, introduced by Cuong[5] in 2014, which is an extension of fuzzy and intuitionistic fuzzy set theory similar to that of neutrosophic sets, which has got a plenty of real time applications, which motivated us to do some contribution in this field of research. The universal set X and $\emptyset$ forms a topology (Munkrer [6]). Popa [18] introduced minimal structures and defined separation axioms using minimal structure. M. Alimohammady, M. Roohi[2] introduced fuzzy minimal structure in lowen sense. S.Bhattacharya (Halder)[3] presented the concept of intuitionistic fuzzy minimal space.

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In section 1, the basic definitions are presented which are useful for our paper and in section 2, the basic definition of picture fuzzy minimal structure space are presented. Further in section 4, some properties of picture fuzzy minimal structure space are also investigated. Finally, in section 5, we introduced an algorithm to solve some applications of picture fuzzy minimal structure space. Section 6 is the conclusion which gives a clear idea about the future research scope in this area.

2. Preliminaries

In this section, we presented the basic definitions developed by [5,10].

Definition 2.1. [5] A picture fuzzy set(in short pfs) $\mathfrak{U}$ on a set $X \neq \emptyset$ is defined by $\mathfrak{U} = \{\langle \xi, m_{\mathfrak{U}}(\xi), a_{\mathfrak{U}}(\xi), n_{\mathfrak{U}}(\xi)\rangle : \xi \in X\}$ where $m_{\mathfrak{U}}, a_{\mathfrak{U}}, n_{\mathfrak{U}} : X \to [0,1]$, denotes the membership of an object, neutral and negative-membership of an object, for each $\xi \in X$ to $\mathfrak{U}$, respectively and $m_{\mathfrak{U}}(\xi) + a_{\mathfrak{U}}(\xi) + n_{\mathfrak{U}}(\xi) \leq 1$ for each $\xi \in X$.

Definition 2.2. [5] Let $\mathfrak{U} = \{\langle \xi, m_{\mathfrak{U}}(\xi), a_{\mathfrak{U}}(\xi), n_{\mathfrak{U}}(\xi)\rangle : \xi \in X\}$ be a picture fuzzy set.

(i) A picture fuzzy set $\mathfrak{U}$ is an empty set i.e., $\mathfrak{U} = 0_{\sim}$ if 0 is membership of an object and 1 is an indeterminacy and non-membership of an object respectively. i.e., $0_{\sim} = \{\xi, (0,0,1) : \xi \in X\}$

(ii) A picture fuzzy set $\mathfrak{U}$ is a universal set i.e., $\mathfrak{U} = 1_{\sim}$ if 1 is membership of an object and 0 is an indeterminacy and non-membership of an object respectively. $1_{\sim} = \{\xi, (1,0,0) : \xi \in X\}$

(iii) $\mathfrak{U}_1 \cup \mathfrak{U}_2 = \{\xi, max\{m_{\mathfrak{U}_1}(\xi), m_{\mathfrak{U}_2}(\xi)\}, min\{a_{\mathfrak{U}_1}(\xi), a_{\mathfrak{U}_2}(\xi)\}, min\{n_{\mathfrak{U}_1}(\xi), n_{\mathfrak{U}_2}(\xi)\} : \xi \in X\}$

(iv) $\mathfrak{U}_1 \cap \mathfrak{U}_2 = \{\xi, min\{m_{\mathfrak{U}_1}(\xi), m_{\mathfrak{U}_2}(\xi)\}, max\{a_{\mathfrak{U}_1}(\xi), a_{\mathfrak{U}_2}(\xi)\}, max\{n_{\mathfrak{U}_1}(\xi), n_{\mathfrak{U}_2}(\xi)\} : \xi \in X\}$

(v) $\mathfrak{U}_1^C = \{\xi, n_{\mathfrak{U}}(\xi), a_{\mathfrak{U}}(\xi), m_{\mathfrak{U}}(\xi) : \xi \in X\}$

Definition 2.3. [10] A picture fuzzy topology (PFT) in Thao sense on a nonempty set X is a family τ of NSs in X satisfying three axioms:

(1) Empty set $0_{\sim}$ and universal set $1_{\sim}$ are members of τ.
(2) $\mathfrak{U}_1 \cap \mathfrak{U}_2 \in \tau$ where $\mathfrak{U}_1, \mathfrak{U}_2 \in \tau$.
(3) $\bigcup_{i=1}^{\infty} \mathfrak{U}_i \in \tau$ where each $\mathfrak{U}_i \in \tau$.

Each picture fuzzy sets in picture fuzzy topological spaces are called picture fuzzy open sets. Its complements are called picture fuzzy closed sets.

Definition 2.4. [10] Let PFS $\mathfrak{U}$ in PFTS X. Then a picture fuzzy interior of $\mathfrak{U}$ and a picture fuzzy closure of $\mathfrak{U}$ are defined by

pf-int($\mathfrak{U}$) = max $\{F : F$ is a picture fuzzy open set in X and $F \leq \mathfrak{U}\}$ and

pf-cl($\mathfrak{U}$) = min $\{F : F$ is a picture fuzzy closed set in X and $F \geq \mathfrak{U}\}$ respectively.

3. Picture Fuzzy Minimal Structure

Definition 3.1. Let the picture fuzzy minimal structure space over a universal set X be denoted by pf_m. pf_m is said to be picture fuzzy minimal structure space over X if it satisfying following the axiom:

(1) $0_\sim, 1_\sim \in pf_m$.

A family of picture fuzzy minimal structure space is denoted by (X, pf_{mX})
Note that picture fuzzy empty set and picture fuzzy universal set can form a topology and it is known as picture fuzzy minimal structure space.

Definition 3.2. Each picture fuzzy set in picture fuzzy minimal structure space is picture fuzzy minimal open set.
The complement of picture fuzzy minimal open set is picture fuzzy minimal closed set.

Remark 3.3. Every picture fuzzy open set is a picture fuzzy minimal open set and every picture fuzzy minimal closed set is a picture fuzzy minimal closed set.

Therefore basic operations such as union, intersection, complement of picture fuzzy minimal sets are same as picture fuzzy sets. In this paper, we refer Definition 2.2 for basic operations.

Examples. We know that $0_\sim = \{\xi, (0,0,1)\} \& 1_\sim = \{\xi, (1,0,0)\}$ are picture fuzzy minimal open sets. Lets find out their complements.
$0_\sim^C = \{\xi, (1,0,0)\} = 1_\sim$ and $1_\sim^C = \{\xi, (0,0,1)\} = 0_\sim$. This clears that $0_\sim$ and $1_\sim$ are both picture fuzzy minimal open and closed set.

Remark 3.4. From Definition 3.2. the following is obvious

(1) Every picture fuzzy topological spaces are picture fuzzy minimal structure space but converse is not true.

The following Example 3.1 proves the above Remark 3.2.

Examples. Let $A = \{< 0.5, 0.4, 0.1 >: \xi\}$, $B = \{< 0.6, 0.2, 0.1 >: \xi\}$ are picture fuzzy sets over the universal set $X = \{\xi\}$. Then the picture fuzzy minimal structure space is $pf_m = \{0, 1, A, B\}$. But pf_m is not a picture fuzzy topological space, since arbitrary union and finite intersection doesn't hold in pf_m.

Definition 3.5. A is pf_m-closed if and only if $pf_m cl(A) = A$.
Similarly, A is a pf_m-open if and only if $pf_m int(A) = A$.

Definition 3.6. Let pf_m be any picture fuzzy minimal structure space and A be any picture fuzzy set. Then

(1) Every $A \in pf_m$ is open and its complement is closed.
(2) pf_m-closure of $A = min\{F : F$ is a picture fuzzy minimal closed set and $F \geq A\}$ and it's denoted by $pf_m cl(A)$.
(3) pf_m-interior of $A = max\{F : F$ is a picture fuzzy minimal open set and $F \leq A\}$ and it is denoted by $pf_m int(A)$.

In general $pf_m int(A)$ is subset of A and A is a subset of $pf_m cl(A)$.

Theorem 3.7. Suppose A and B are any two picture fuzzy minimal sets of picture fuzzy minimal structure space pf_m over X. Then

(i) $pf_m^C = \{0, 1, A_i^C\}$ where A_i^C is a picture fuzzy complement of a set A_i.
(ii) $X - pf_m int(B) = pf_m cl(X - B)$.

(iii) $X - pf_m cl(B) = pf_m int(X - B)$.

(iv) $pf_m cl(A^C) = (pf_m cl(A))^C = pf_m int(A)$.

(v) pf_m closure of an empty set is an empty set and pf_m closure of a universal set is a universal set. Similarly, pf_m interior of an empty set and universal set respectively an empty and a universal set.

(vi) If B is a subset of A then $pf_m cl(B) \leq pf_m cl(A)$ and $pf_m int(B) \leq pf_m int(A)$.

(vii) $pf_m cl(pf_m cl(A)) = pf_m cl(A)$ and $pf_m int(pf_m int(A)) = pf_m int(A)$.

(viii) $pf_m cl(A \vee B) = pf_m cl(A) \vee pf_m cl(B)$

(ix) $pf_m cl(A \wedge B) = pf_m cl(A) \wedge pf_m cl(B)$

Proof:
(i) We know that $A^C = X - A$. Then $pf_m cl(X - A) = pf_m cl(A^C) = (pf_m cl(A))^C = pf_m int(A)$, from (iv).
Similarly for (ii).
(vi) Let $B \leq A$. We know that $B \leq pf_m cl(B)$ and $A \leq pf_m cl(B)$. So $B \leq pf_m cl(B) \leq A \leq pf_m cl(A)$. Therefore $pf_m cl(B) \leq pf_m cl(A)$.
Proof of (vii) is straight forward.
(viii) We know that $A \leq A \vee B$ and $B \leq A \vee B$. $pf_m cl(A) \leq pf_m cl(A \vee B)$ and $pf_m cl(B) \leq pf_m cl(A \vee B)$ this implies $pf_m cl(A) \vee pf_m cl(B) \leq pf_m cl(A \vee B). \longrightarrow (*)$
Also $A \leq pf_m cl(A) and B \leq pf_m cl(B) \Rightarrow A \vee B \leq pf_m cl(A) \vee pf_m cl(B)$. $pf_m(A \vee B) \leq pf_m(pf_m cl(A) \vee pf_m cl(B)) = pf_m cl(A) \vee pf_m cl(B) \longrightarrow (**)$.
From (*) and (**), we have $pf_m cl(A \vee B) = pf_m cl(A) \vee pf_m cl(B)$.

Examples. Consider Example 3.2, the complement of pf_m is $\{0, 1, A^C, B^C\}$ where $A^C = \{< 0.1, 0.4, 0.5 > /\xi : \xi \in X\}$ and $B^C = \{< 0.1, 0.6, 0.2 > /\xi : \xi \in X\}$.

Definition 3.8. A function $f : (X, pf_{mX}) \to (Y, pf_{mY}))$ is called picture fuzzy minimal continuous function if and only if $f^{-1}(V) \in pf_{mX}$ whenever $V \in pf_{mY}$.

Definition 3.9. Boundary of a picture fuzzy minimal set A (in short $Bd(A)$) of picture fuzzy minimal structure (X, pf_{mX}) is the intersection of $pf_m closure$ of the set A and $pf_m closure$ of $X - A$.
i.e., $Bd(A) = pf_m cl(A) \cap pf_m cl(X - A)$

Theorem 3.10. If $(X, pf_{mX})$and (Y, pf_{mY}) are picture fuzzy minimal structure space. Then

(1) Identity map from (X, pf_{mX}) to (Y, pf_{mY}) is a picture fuzzy minimal continuous function.
(2) Any constant function which maps from (X, pf_{mX}) to (Y, pf_{mY}) is a picture fuzzy minimal continuous function.

Theorem 3.11. Let the map f from picture fuzzy minimal structure space (X, pf_{mX}) to picture fuzzy minimal structure space (Y, pf_{mY}). Then the following are equivalent,

(1) The map f is a picture fuzzy minimal continuous function.
(2) $f^{-1}(V)$ is a picture fuzzy minimal closed set for each picture fuzzy minimal closed set $V \in pf_{mY}$.

(3) $pf_m cl(f^{-1}(V)) \leq f^{-1}(pf_m cl(V))$, for each $V \in pf_{mY}$.
(4) $pf_m cl(f(A)) \geq f(pf_m cl(A))$, for each $A \in pf_{mX}$.
(5) $pf_m int(f^{-1}(V)) \geq f^{-1}(pf_m int(V))$, for each $V \in pf_{mX}$.

Proof.
(1) $\rightarrow$ (2): Let A be a pf_m-closed in Y. Then $f^{-1}(A)^C = f^{-1}(A^C) \in pf_{mX}$.
(2) $\rightarrow$ (3):$pf_m cl(f^{-1}(A) = \wedge\{D : f^{-1}(A) \leq D, D^C \in pf_{mX}\} \leq \wedge\{f^{-1}(D) : A \leq D, D^C \in pf_{mY}\} = f^{-1}(\{D : A \leq D^C \in pf_{mY}\}) = F^{-1}(pf_m cl(A))$.
(3) $\rightarrow$ (4): Since $A \leq f^{-1}(f(A))$, then $pf_m cl(A) \leq pf_m cl f^{-1}(f(A)) \leq f^{-1}(pf_m cl(f(A)))$. Therefore$f(pf_m cl(A)) \leq pf_m cl(f(A))$.
(4) $\rightarrow$ (5): $f(pf_m int f^{-1}(A))^C = f(pf_m cl(f^{-1}$
$(A))^C) = f(pf_m cl(f(A)^C)) \leq pf_m cl(f(f^{-1}(A^C))) \leq pf_m cl(A^C) = (pf_m int(A))^C$. This implies that $pf_m int(f_{-1}(B))^C \leq f^{-1}(pf_m int(A))^C = (f^{-1}(pf_m int(A)))^C$.
Taking complement on both sides, $f^{-1}(pf_m int(A)) \leq pf_m int f^{-1}(B)$.

Definition 3.12. Let (X, pf_{mX}) be picture fuzzy minimal structure space.

i. Arbitrary union of picture fuzzy minimal open sets in (X, pf_{mX}) is picture fuzzy minimal open. (Union Property)
ii. Finite intersection of picture fuzzy minimal open sets in (X, pf_{mX}) is picture fuzzy minimal open. (intersection Property)

4. Picture Fuzzy Minimal Subspace

In this section, we introduced the picture fuzzy minimal subspace and investigate some properties of subspace.

Definition 4.1. Let A be a picture fuzzy set in picture fuzzy minimal structure space (X, pf_{mX}). Then Y is said to be picture fuzzy minimal subspace if $(Y, pf_{mY}) = \{A \cap U : U \in pf_{mY}\}$

Lemma 4.2. *If picture fuzzy set b in the basis B for picture fuzzy minimal structure space X. Then the collection $B_Y = \{b \cap Y : Y \subset X\}$ is a basis for picture fuzzy minimal subspace on Y.*

Proof.
Given a picture fuzzy set A in X and C is a picture fuzzy set in both A and subset Y of X. Consider a basis element b of B such that b in C and in Y. Then $C \in B \cap Y \subset U \cap Y$. Hence B_Y is a basis for the picture fuzzy minimal subspace on the set Y.

Lemma 4.3. *Let (Y, pf_{mY}) be a subspace of (X, pf_{mX}). If A is a picture fuzzy set in Y and $Y \subset X$, then A is in (X, pf_{mX}).*

Proof:
Given that picture fuzzy set A in (Y, pf_{mY}). $A = Y \cap B$ for some picture fuzzy set $B \in X$. Since Y and B in X. Then A is in X.

Theorem 4.4. Suppose (Y, pf_{mY}) is a picture fuzzy minimal subspace of (X, τ_X).

(1) If the picture fuzzy minimal structure space (X, pf_{mX}) has the union property, then the subspace (Y, pf_{mY}) also has union property.

(2) If the picture fuzzy minimal structure space (X, pf_{mX}) has the intersection property, then the subspace (Y, pf_{mY}) also has union property.

Proof.
Suppose the family of open set $V_i : i \in Y$ in picture fuzzy minimal subspace(Y, pf_{mY}) then there exist a family of open sets $\{U_j : j \in X\}$ in picture fuzzy minimal structure space (X, pf_{mX}) such that $V_i = U_j \cap A, \forall i \in Y$ where $A \in pf_{mY}$. $\bigcup_{(i \in Y)} V_i = \bigcup_{(j \in X)} (U_j \cap A) = \bigcup_{(i \in Y)} U_j \cap A$. Since (X, pf_{mX}) has union property then (Y, pf_{mY}) also has union property. The proof of (ii) is similarly to (i).

Definition 4.5. Suppose (B, pf_{mB}) and (C, pf_{mC}) are picture fuzzy minimal subspaces of picture fuzzy minimal structure spaces (Y, pf_{mY}) and (Z, pf_{mZ}) respectively. Also, suppose that f is a mapping from (Y, pf_{mY}) to (Z, pf_{mZ}) is a mapping. We say that f is a mapping from (B, pf_{mB}) into (C, pf_{mC}) if the image of B under f is a subset of C.

Definition 4.6. Suppose (A, pf_{mA}) and (B, pf_{mB}) are picture fuzzy minimal subspaces of picture fuzzy minimal structure spaces (Y, pf_{mY}) and (Z, pf_{mZ}) respectively. The mapping f from (A, pf_{mA}) into (B, pf_{mB}) is called a

(i) comparative picture fuzzy minimal continuous, if $f^1(W) \wedge A \in pf_{mA}$ for every picture fuzzy minimal structure set W in B,
(ii) comparative picture fuzzy minimal open, if $f(V) \in pf_{mB}$ for every fuzzy set $V \in pf_{mA}$.

5. Application of picture fuzzy minimal structure space

The application of picture fuzzy minimal structure space is based on the minimal element and maximal element. In picture fuzzy minimal structure space, $0_\sim$ is the minimal element and $1_\sim$ is the maximal element. The application of picture fuzzy minimal structure space used in consumer theory where the customer has only two main objectives, one is to maximize the quantity or maximize the durability, and the other is to minimize purchase cost.
The following algorithm is proposed to take optimal decision.

5.1 Proposed algorithm for optimal decision making using PFMS

Step 1.

X/E	β_1	β_2	β_3	....	β_n
α_1	γ_{11}	γ_{12}	γ_{13}	....	γ_{1n}
α_2	γ_{21}	γ_{22}	γ_{23}	....	γ_{2n}
.	.	.	.	.	.
α_m	γ_{m1}	γ_{m2}	γ_{m3}	....	γ_{mn}

Input m Attributes and n alternatives.
Step 2. Construct the picture fuzzy minimal structure from the data. $\tau_k = \{0_\sim, 1_\sim, U_k\}$ where $U_k = \{\gamma_{1k}, \gamma_{2k} .... \gamma_{mk}\}$
Step 3. compute the picture fuzzy score function (in short, NF) using the following simple formula, $NF(U_k) = \frac{1}{3m}[\sum_{i=1}^{m}[1 + m_i - a_i - n_i]]$
Step 4. Arrange the score function $\mathfrak{U}_k$ which we calculated in step 3 in ascending order. Choose the largest score value $\mathfrak{U}_k$ for better decision.

6. Numerical Example

Let's consider the following example. Let the set of three brands of bikes be $X = \{\mathfrak{B}_1, \mathfrak{B}_2, \mathfrak{B}_3\}$ and the parameter set $E = \{\zeta_1, \zeta_2, \zeta_3\}$, where ζ_1= cost, ζ_2 = safety, and ζ_3= maintenance of the bike. A customer will assign minimum value of $0_\sim$ to bad features,maximum $1_\sim$ to the best feature of the product. Membership, neutrality and negative-membership values taken from customer's review rating. Membership referred to cost of the bike is worth to the model, safe and low maintenance cost. negative-membership referred to cost of the bike is not worth to the model, not safe due to break failure or some other reason and high maintenance cost. Neutral referred to neutrality of cost of the bike, safe if drive safe and maintenance is neutral.

X/E	$\mathfrak{B}_1$	$\mathfrak{B}_2$	$\mathfrak{B}_3$
ζ_1	(0.4,0.3,0.2)	(0.5,0.1,0.2)	(0.6,0.3,0.1)
ζ_2	(0.6,0.2,0.1)	(0.4,0.1,0.4)	(0.7,0.1,0.2)
ζ_3	(0.5,0.1,0.3)	(0.6,0.3,0.1)	(0.4,0.3,0.2)

Let us assume the values are taken from customer review rating for the models $\mathfrak{B}_1, \mathfrak{B}_2$ and $\mathfrak{B}_3$ with parameters ζ_1, ζ_2 and ζ_3.
Step 2. The picture fuzzy minimal structure
$\tau_1 = \{0_\sim, 1_\sim, U_1\}$ where $\mathfrak{U}_1 = \{(0.4, 0.3, 0.2), (0.5, 0.1, 0.2), (0.6, 0.3, 0.1)\}$
Similarly, $\tau_2 = \{0_\sim, 1_\sim, U_2\}$ where $\mathfrak{U}_2 = \{(0.6, 0.2, 0.1), (0.4, 0.1, 0.4), (0.7, 0.1, 0.2)\}$
$\tau_3 = \{0_\sim, 1_\sim, U_3\}$ where $\mathfrak{U}_3 = \{(0.5, 0.1, 0.3), (0.6, 0.3, 0.1), (0.4, 0.3, 0.2)\}$
Step 3. picture fuzzy score functions are
$PFS(\mathfrak{U}_1) = 0.3667$
$PFS(\mathfrak{U}_2) = 0.4$
$PFS(\mathfrak{U}_3) = 0.356$
Step 4. The picture fuzzy score functions are arranged in ascending order as follows $\mathfrak{U}_3 \leq \mathfrak{U}_1 \leq \mathfrak{U}_2$. Based on score function, $\mathfrak{U}_2$ is the largest score function. $\mathfrak{U}_2$ related to the model $\mathfrak{B}_2$. Hence Model $\mathfrak{B}_2$ is best bike model to buy.

6. Conclusion

In this paper, picture fuzzy minimal structure space is introduced and some of its properties investigated along with this. picture fuzzy minimal continuous and subspace are also investigated with few properties. Finally, application of picture fuzzy minimal structure is discussed. Future work of this paper is to investigate and study various open sets and separation axioms in picture fuzzy minimal structure space. Also the application part discussed in this work leads to analyze in weak structure.

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