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# $N_\delta$-CONTINUITY IN NEUTROSOPHIC TOPOLOGICAL SPACES

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***Abstract.*** The research work proposes the theory of $N_\delta$-continuous functions(briefly $N_{\delta-CF}$) and neutrosophic almost continuous functions(briefly $N_{ACF}$) in neutrosophic topological spaces(briefly $N_{TS}$). The characterizations of $N_{\delta-CF}$ and $N_{ACF}$ are analyzed and the properties are inspected.

## 1. Introduction

In 1965 Zadeh[10] with his vital theory on fuzzy sets ($FS$) that has non-probabilistic uncertainty into the mathematical pattern where the said concept never gets exhorted by the randomness of an event. A degree of belongingness with some value prevails between 0 and 1 for each and every element in this, universe which is the membership value of an element in that set, which is known as $FS$, to analyze vague data in applications. As to handle uncertain circumstances which up vote from the prevailing vagueness or from a partial belongingness, $FS$ is highly likely to handle successfully. Although, modelling a mathematical element is uncertain for every kind of real physical problems.

In 1986 Atanassov[8], generalized $FS$ which in turn led to the emergence of the concept of intuitionistic fuzzy sets ($IFS$). In $FS$ it is a known fact that it has a membership value for every element based on its vagueness and belongingness. Whereas, Atanassov states that there is a non-membership value devoted to each element. When the sum of membership value and non-membership value is less or equal to unity, a constraint occurs. In $IFS$, degree of non-belongingness is dependent on the degree of belongingness. The $IFS$ by nature has the ability to analyze the complete and incomplete vague data. $IFS$s are resourceful in applications such as expert systems, belief systems and information fusion etc.

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In Intuitionistic Fuzzy Topological Spaces($IFTS$) was introduced by Coker[4] in 1997. In 2012 Eom and Lee[14] introduced the concept of intuitionistic fuzzy $\delta(IF_\delta)$-closure as a generalization of the concept of fuzzy $\delta$-closure by Ganguly and Saha[11] in 1988. Also Eom and Lee[13] introduced and investigated the properties of $IF_\delta$-continuous and intuitionistic fuzzy almost continuous functions(IFACF) in terms of $IF_\delta$-closure and interior.

A theory has been novelized by Samrandache[5], which is a knowledge of impartial thought known as Neutrosophic Logic and sets which represents a distinct character. The logic behind the Neutrosophic sets(briefly $N_S$) is that every elements proposition is calculated to have a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F), which lies between [0, 1]. The indeterminacy factor, is independent of truth and falsity values in a $N_S$. The constraints are nullified among the degree of truth, degree of indeterminacy and degree of falsity. The $N_S$ has a wide range of real time applications in fields such as Applied Mathematics, Artificial Intelligence, Computer Science, decision making, Electrical, Electronics, Information Systems, Mechanics, Medicine and Management Science etc.

F. Smarandache[6], stated that $N_S$ is also a generalization of Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n-HyperSpherical Fuzzy Set, while Neutrosophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision. And Refined Neutrosophic Set (RNS) is an extension of Neutrosophic Set. And all these sets are more general than Intuitionistic Fuzzy Set. The main distinction between Neutrosophic Set $N_S$ and all previous set theories are stuided.

A different new concept of $N_{TS}$ was stated by Salama and Alblowi[1], which is a generalization of Coker's[4] intuitionistic fuzzy topology and Chang's[3] fuzzy topology. Further, Salama et al.[2] introduced The neutrosophic closed sets(briefly $N_{CS}$'s) and neutrosophic continuous functions, Nandhini et .al[12] introduced $N_{\alpha g^{\#}\psi}$-continuous in $N_{TS}$.

As a genralization of $IF_\delta$-closure and interior, Recently Damodharan and Vigneshwaran[9] introduced the concept of $N_\delta$-closure and interior in $N_{TS}$.

The purpose of this paper is to propose the idea of $N_{\delta-CF}$ and $N_{ACF}$ in terms of $N_\delta$-closure and $N_\delta$-interior

## 2. Preliminaries

A novel idea is researched with having the basic concepts of Salame et .al[2] as how the $N_S$ can be defined. Operations such as union, intersection and subsets of two $N_S$'s. The concept of Chang[3], is incorporated to define the complement of $N_S$'s. The basic properties of $N_{TS}$ is adapted for the research work to determine the $N_\delta$-closure and $N_\delta$-interior. To strengthen the research work being carried out, the Ideas and conceptual framework of the previous article Damodharan and Vigneshwaran[9] were unearthed, which involves Neutrosophic regular open q-neighborhood(briefly $N^q_{RO}$-nhd), concept of quasi coincident between $N_P$ and $N_S$ and between two $N_S$'s.

## 3. $N_\delta$-continuous functions

**Definition 3.1.** A $N_S$ P is said to be neutrosophic $\delta$-neighborhood(briefly $N_\delta$-nhd) of a neutrosophic point(briefly $N_P$) $s_{(,\upsilon,\omega,\xi)}$, if there exists $N^q_{RO}$-nhd Q of $s_{(,\upsilon,\omega,\xi)}$ such that $Q \subseteq P$.

**Lemma 3.2.** *A $N_S$ A is an $N_\delta$-open set(briefly $N_{\delta-OS}$) in $(S, \acute{\tau})$ iff for any $N_P$ $s_{(,\upsilon,\omega,\xi)}$ with $s_{(,\upsilon,\omega,\xi)} qA$, A is a $N_\delta$-nhd of $s_{(,\upsilon,\omega,\xi)}$.*

*Proof.* Let A be $N_{\delta-OS}$ in $(S, \acute{\tau})$ such that $s_{(,\upsilon,\omega,\xi)} qA$. Then $s_{(,\upsilon,\omega,\xi)} \nsubseteq A^c$. Since $A^c$ is $N_\delta$-closed set(briefly $N_{\delta-CS}$), we have $s_{(,\upsilon,\omega,\xi)} \notin A^c = Ncl_\delta(A^c)$. Then there exists a $N^q_{RO}$-nhd P of $s_{(,\upsilon,\omega,\xi)}$ such that $P\tilde{q}A^c$. Thus $P \subseteq A$. Hence A is a $N_\delta$-nhd of $s_{(,\upsilon,\omega,\xi)}$. Conversely, to show that $A^c$ is a $N_{\delta-CS}$, take any $s_{(,\upsilon,\omega,\xi)} \notin A^c$. Then we have $s_{(,\upsilon,\omega,\xi)} qA$. Thus A is a $N_\delta$-nhd of $s_{(,\upsilon,\omega,\xi)}$. Therefore there exists a neutrosophic open q(briefly $N^q_O$)-nhd Q of $s_{(,\upsilon,\omega,\xi)}$ such that $Q \subseteq A^c$, i.e. $s_{(,\upsilon,\omega,\xi)} \notin Ncl_\delta(A^c)$. Since $Ncl_\delta(A^c) \subseteq A^c$, we have $A^c$ is a $N_{\delta-CS}$. Hence P is a $N_{\delta-OS}$.

*As to recall the concepts of fuzzy $\delta$-continuous function Ganguly and Saha*[11] *and intiuitionistic fuzzy $\delta$-continuous function of Eom and Lee*[13]*, a similar definition is defined in the $N_{TS}$ as follows:*

**Definition 3.3.** A function $k : (S, \acute{\tau}) \to (T, \acute{\sigma})$ is said to be a $N_{\delta-CF}$ if for each $N_P$ $s_{(,\upsilon,\omega,\xi)}$ in S and for any $N^q_{RO}$-nhd Q of $f\left(s_{(,\upsilon,\omega,\xi)}\right)$, there exists a $N^q_{RO}$-nhd P of $s_{(,\upsilon,\omega,\xi)}$ such that $k(P) \subseteq Q$.

**Theorem 3.4.** *For the function $k : (S, \acute{\tau}) \to (T, \acute{\sigma})$ the following statements are equivalent:*

*(1) k is a $N_{\delta-CF}$.*
*(2) $k(Ncl_\delta(P)) \subseteq Ncl_\delta(k(P))$ for each $N_S$ P in S.*
*(3) $Ncl_\delta\left(k^{-1}(Q)\right) \subseteq k^{-1}(Ncl_\delta(Q))$ for each $N_S$ Q in T.*
*(4) $k^{-1}(Nint_\delta(Q)) \subseteq Nint_\delta\left(k^{-1}(Q)\right)$ for each $N_S$ Q in T.*

*Proof.*

(1) $\Rightarrow$ (2)**:** *Let $s_{(,\upsilon,\omega,\xi)} \in Ncl_\delta(P)$, and let B be a $N^q_{RO}$ q - neighborhood of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$ in T. By (1), there exists a $N^q_{RO}$ q - neighborhood A of $s_{(,\upsilon,\omega,\xi)}$ such that $k(A) \subseteq B$. Since $s_{(,\upsilon,\omega,\xi)} \in Ncl_\delta(P)$ and A is a $N^q_{RO}$-nhd of $s_{(,\upsilon,\omega,\xi)}$, $AqP$. So $k(A)\,qk(P)$. Since $k(A) \subseteq B$, $Bqk(P)$. Then $k\left(s_{(,\upsilon,\omega,\xi)}\right) \in Ncl_\delta(k(P))$. Hence $k(Ncl_\delta(P)) \subseteq Ncl_\delta(k(P))$.*

(2) $\Rightarrow$ (3)**:** *Let Q be a $N_S$ in T. Then $k^{-1}(Q)$ is a $N_S$ in S. By (2), $k\left(Ncl_\delta\left(k^{-1}(Q)\right)\right) \subseteq Ncl_\delta\left(k\left(k^{-1}(Q)\right)\right) \subseteq Ncl_\delta(Q)$. Thus $Ncl_\delta\left(k^{-1}(Q)\right) \subseteq k^{-1}(Ncl_\delta(Q))$.*

(3) $\Rightarrow$ (1)**:** *Let $s_{(,\upsilon,\omega,\xi)}$ be a $N_P$ in S, and let Q be a $N^q_{RO}$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$ in T. Since $Q^c$ is a neutrosophic regular closed set (briefly $N_{RCS}$), $Q^c$ is a neutrosophic semi-open set. By Theorem 3.3* [9]*, $Ncl(Q^c) = Ncl_\delta(Q^c)$. Therefore $s_{(,\upsilon,\omega,\xi)} \notin k^{-1}(Ncl_\delta(Q^c))$. By (3), $s_{(,\upsilon,\omega,\xi)} \notin Ncl_\delta\left(k^{-1}(Q^c)\right)$. Then there exists a $N^q_{RO}$-nhd P of $s_{(,\upsilon,\omega,\xi)}$ such that $P\tilde{q}k^{-1}(Q^c) = \left(k^{-1}(Q)\right)^c$. So $P \subseteq k^{-1}(Q)$, i.e. $k(P) \subseteq Q$. Hence k is a $N_{\delta-CF}$).*

(3) $\Rightarrow$ (4)**:** *Let Q be a $N_S$ in T. By (3), $Ncl_\delta\left(k^{-1}(Q^c)\right) \subseteq k^{-1}(Ncl_\delta(Q^c))$. Thus $k^{-1}(Nint_\delta(Q)) = k^{-1}\left((Ncl_\delta(Q^c))^c\right) = \left(k^{-1}(Ncl_\delta((Q^c)))\right)^c \subseteq$*

$\left(Ncl_\delta\left(k^{-1}\left(Q^c\right)\right)\right)^c =$
$\left(Ncl_\delta\left(\left(k^{-1}\left(Q\right)\right)^c\right)\right)^c = Nint_\delta\left(k^{-1}\left(Q\right)\right).$

$(4) \Rightarrow (3)$: *Let $Q$ be a $N_S$ in $T$. Then $Q^c$ is a $N_S$ in $T$. By the hypothesis,* $k^{-1}\left(Nint_\delta\left(Q^c\right)\right) \subseteq Nint_\delta\left(k^{-1}\left(Q^c\right)\right)$. *Thus* $Ncl_\delta\left(k^{-1}\left(Q\right)\right) = \left(Nint_\delta\left(\left(k^{-1}\left(Q\right)\right)^c\right)\right)^c =$
$\left(Nint_\delta\left(k^{-1}\left(Q^c\right)\right)\right)^c \subseteq \left(k^{-1}\left(Nint_\delta\left(Q^c\right)\right)\right)^c = k^{-1}\left(\left(Nint_\delta\left(Q^c\right)\right)^c\right) = k^{-1}\left(Ncl_\delta\left(Q\right)\right).$
*Hence* $Ncl_\delta\left(k^{-1}\left(Q\right)\right) \subseteq k^{-1}\left(Ncl_\delta\left(Q\right)\right)$.

*The $N_{\delta-CF}$ is characterized in terms of $N_\delta$-open and $N_{\delta-CS}$s.*

**Theorem 3.5.** *For the function $k : (S, \acute{\tau}) \rightarrow (T, \acute{\sigma})$ the following statements are equivalent:*

*(1) $k$ is a $N_{\delta-CF}$.*
*(2) $k^{-1}(A)$ is a $N_{\delta-CS}$ for each $N_{\delta-CS}$ $A$ in $S$.*
*(3) $k^{-1}(A)$ is a $N_{\delta-OS}$ for each $N_{\delta-OS}$ $A$ in $S$.*

*Proof.*

$(1) \Rightarrow (2)$: *Let $A$ be a $N_{\delta-CS}$ in $S$. Then $A = Ncl_\delta(A)$. By Theorem 3.4,* $Ncl_\delta\left(k^{-1}(A)\right) \subseteq k^{-1}\left(Ncl_\delta(A)\right) = k^{-1}(A)$. *Hence* $k^{-1}(A) = Ncl_\delta\left(k^{-1}(A)\right)$. *Therefore $k^{-1}(A)$ is a $N_{\delta-CS}$.*

$(2) \Rightarrow (3)$: *Let $A$ be a $N_{\delta-OS}$ in $S$. Then $Nint_\delta(A) = A$, we know that* $Nint_\delta\left(k^{-1}(A)\right) = \left(Ncl_\delta\left(k^{-1}(A)\right)^c\right)^c$. *By using* (2) $\left(Ncl_\delta\left(k^{-1}(A)\right)^c\right)^c = \left(\left(k^{-1}(A)\right)^c\right)^c = k^{-1}(A)$. *Therefore* $Nint_\delta\left(k^{-1}(A)\right) = k^{-1}(A)$. *Hence $k^{-1}(A)$ is a $N_{\delta-OS}$ in $S$.*

$(3) \Rightarrow (1)$: *Let $s_{(,\upsilon,\omega,\xi)}$ be a $N_P$ in $S$, and let $Q$ be a $N_O^q$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. By Corollary 3.3 of* [9], *$Q$ is an $N_{\delta-OS}$. By the hypothesis, $k^{-1}(Q)$ is an $N_{\delta-OS}$. since $s_{(,\upsilon,\omega,\xi)} q k^{-1}(Q)$, By Lemma 3.2, we have that $k^{-1}(Q)$ is an $N_\delta$-nhd of $s_{(,\upsilon,\omega,\xi)}$. Therefore, there exists a $N_{RO}^q$-nhd $P$ of $s_{(,\upsilon,\omega,\xi)}$ such that $P \subseteq k^{-1}(Q)$. Hence $k(P) \subseteq Q$.*

*The $N_{\delta-CF}$ is characterized in terms of $N_\delta$-nhds.*

**Theorem 3.6.** *A function $k : (S, \acute{\tau}) \rightarrow (T, \acute{\sigma})$ is $N_{\delta-CF}$ iff for each $N_P$ $s_{(\alpha,\beta,\lambda)}$ of $S$ and each $N_\delta$-nhd $N$ of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$, the $N_S$ $k^{-1}(N)$ is an $N_\delta$-nhd of $s_{(,\upsilon,\omega,\xi)}$.*

*Proof. Let $s_{(,\upsilon,\omega,\xi)}$ be a $N_P$ in $S$, and let $N$ be an $N_\delta$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. Then there exists a $N_{RO}^q$-nhd $Q$ of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$ such that $Q \subseteq N$. Since $k$ is an $N_{\delta-CF}$, there exists a $N_{RO}^q$-nhd $P$ of $s_{(,\upsilon,\omega,\xi)}$ such that $k(P) \subseteq Q$. Thus, $P \subseteq k^{-1}(Q) \subseteq N$. Hence $k^{-1}(N)$ is an $N_\delta$-nhd of $s_{(,\upsilon,\omega,\xi)}$.*

*Conversely, let $s_{(,\upsilon,\omega,\xi)}$ be an neutrosopic point in $S$, and $Q$ a $N_O^q$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. Then $Q$ is an $N_\delta$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. By the hypothesis, $k^{-1}(Q)$ is an $N_\delta$-nhd of $s_{(,\upsilon,\omega,\xi)}$. By the definition of $N_\delta$-nhd, there exists a $N_{RO}^q$-nhd $P$ of $s_{(,\upsilon,\omega,\xi)}$ such that $P \subseteq k^{-1}(Q)$. Thus $k(P) \subseteq Q$. Hence $k$ is an $N_{\delta-CF}$.*

**Theorem 3.7.** *Let $k : (S, \acute{\tau}) \rightarrow (T, \acute{\sigma})$ be a bijective function, then the following statements are equivalent:*

*(1) $k$ is an $N_{\delta-CF}$.*
*(2) $Nint_\delta(k(P)) \subseteq k(Nint_\delta(P))$ for each $N_S$ $P$ in $S$.*

*Proof.*

$(1) \Rightarrow (2)$**:** *Let $P$ be a $N_S$ in $S$. Then $k(P)$ is a $N_S$ in $T$. By Theorem 3.4,* $k^{-1}(Nint_\delta(k(P))) \subseteq Nint_\delta\left(k^{-1}(k(P))\right)$. *Since $k$ is one-to-one,* $k^{-1}(Nint_\delta(k(P))) \subseteq Nint_\delta\left(k^{-1}(k(P))\right) = Nint_\delta(P)$. *Since $k$ is onto,* $Nint_\delta(k(P)) = k\left(k^{-1}(Nint(k(P)))\right) \subseteq k(Nint(P))$.

$(2) \Rightarrow (1)$**:** *Let $Q$ be a $N_S$ in $T$. Then $k^{-1}(Q)$ is a $N_S$ in $S$. By the hypothesis,* $Nint_\delta\left(k\left(k^{-1}(Q)\right)\right) \subseteq k\left(Nint_\delta\left(k^{-1}(Q)\right)\right)$. *Since $k$ is onto,* $Nint_\delta(Q) = Nint_\delta\left(k\left(k^{-1}(Q)\right)\right) \subseteq k\left(Nint_\delta\left(k^{-1}(Q)\right)\right)$. *Since $k$ is one-to-one,* $k^{-1}(Nint_\delta(Q)) = k^{-1}\left(k\left(Nint_\delta\left(k^{-1}(Q)\right)\right)\right) \subseteq Nint_\delta\left(k^{-1}(Q)\right)$

*Hence by Theorem 3.4, $k$ is an $N_{\delta-CF}$.*

## 4. Neutrosophic almost continuous functions

As we recall the concept of Eom and Lee[13] $IFACF$ and define a similar definition in the $N_{TS}$ as follows.

**Definition 4.1.** A function $k : (S, \acute{\tau}) \to (T, \acute{\sigma})$ is said to be $N_{ACF}$ if for any neutrosophic regular open set (briefly $N_{ROS}$) Q in T, $k^{-1}(Q)$ is a neutrosophic open set(briefly $N_{OS}$ in S.

**Theorem 4.2.** *A function $k : (S, \acute{\tau}) \to (T, \acute{\sigma})$ is a $N_{ACF}$ iff for each $N_P$ $s_{(,\upsilon,\omega,\xi)}$ in $S$ and for any $N_O^q$-nhd $q$ of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$, there exists a $N_O^q$-nhd $P$ of $s_{(,\upsilon,\omega,\xi)}$ such that $k(P) \subseteq Nint(Ncl(Q))$.*

*Proof.* *Let $s_{(,\upsilon,\omega,\xi)}$ be a $N_P$ in $S$ and for any $N_O^q$-nhd $Q$ of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$ there exists $Q \in T$ such that $k\left(s_{(,\upsilon,\omega,\xi)}\right) \in Q$. Hence $s_{(,\upsilon,\omega,\xi)} \in k^{-1}(Q)$. Since $Q \in T$ then $Q \subseteq Nint(Ncl(Q))$ and $k^{-1}(Q) \subseteq k^{-1}(Nint(Ncl(Q)))$. Thus $s_{(,\upsilon,\omega,\xi)} \in k^{-1}(Nint(Ncl(Q)))$. Since $Nint(Ncl(Q))$ is a $N_{OS}$ then $k^{-1}(Nint(Ncl(Q)))$ is a $N_{OS}$ in $S$ and $s_{(,\upsilon,\omega,\xi)} \in k^{-1}(Nint(Ncl(Q)))$. Consequently, there exists $P \in S$ such that $s_{(,\upsilon,\omega,\xi)} \in P \subseteq k^{-1}(Nint(Ncl(Q)))$ Hence $k(P) \subseteq Nint(Ncl(Q))$.*

**Theorem 4.3.** *For the function $k : (S, \acute{\tau}) \to (T, \acute{\sigma})$ the following statements are equivalent:*

*(1)* $k$ *is a* $N_{ACF}$.
*(2)* $k(Ncl(P)) \subseteq Ncl_\delta(k(P))$ *for each* $N_S$ $P$ *in* $S$.
*(3)* $k^{-1}(Q)$ *is a* $N_{CS}$ *in* $S$ *for each* $N_{\delta-CS}$ $Q$ *in* $T$.
*(4)* $k^{-1}(Q)$ *is a* $N_{OS}$ *in* $S$ *for each* $N_{\delta-OS}$ $Q$ *in* $T$.

*Proof.*

$(1) \Rightarrow (2)$**:** *Let $s_{(,\upsilon,\omega,\xi)} \in Ncl(P)$. Suppose that $k\left(s_{(,\upsilon,\omega,\xi)}\right) \notin Ncl_\delta(k(P))$, then there exists a $N_O^q$-nhd $Q$ of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$ such that $Q\tilde{q}k(P)$. Since $k$ is a $N_{ACF}$, $k^{-1}(Q)$ is a $N_{OS}$ in $S$. Since $Qqk\left(s_{(,\upsilon,\omega,\xi)}\right)$, we have $k^{-1}(Q)\, qs_{(,\upsilon,\omega,\xi)}$. Thus $k^{-1}(Q)$ is a $N_O^q$-nhd of $s_{(,\upsilon,\omega,\xi)}$. Since $s_{(,\upsilon,\omega,\xi)} \in Ncl(P)$, by Theorem 3.1*[**?**]*, we have $k^{-1}(Q)\, qP$. Thus $k\left(k^{-1}(Q)\right) qk(P)$. Since $k\left(k^{-1}(Q)\right) \subseteq Q$, we have $Qqk(P)$. This is a contradiction. Hence $k(Ncl(P)) \subseteq Ncl_\delta(k(P))$.*

- $(2) \Rightarrow (3)$**:** *Let $Q$ be an $N_{\delta-CS}$ in $T$. Then $k^{-1}(Q)$ is a $N_S$ in $S$. By the hypothesis, $k\left(Ncl\left(k^{-1}(Q)\right)\right) \subseteq Ncl_\delta\left(k\left(k^{-1}(Q)\right)\right) \subseteq Ncl_\delta(Q) = Q$. Thus $Ncl\left(k^{-1}(Q)\right) \subseteq k^{-1}(Q)$. Hence $k^{-1}(Q)$ is a $N_{CS}$ in $S$.*
- $(3) \Rightarrow (4)$**:** *Let $Q$ be an $N_{\delta-OS}$ in $T$. Then $Q^c$ is an $N_{\delta-CS}$ in $T$. By the hypothesis, $k^{-1}(Q^c) = \left(k^{-1}(Q)\right)^c$ is a $N_{CS}$ in $S$. Hence $k^{-1}(Q)$ is a $N_{OS}$ in $S$.*
- $(4) \Rightarrow (1)$**:** *Let $s_{(,\upsilon,\omega,\xi)}$ be a $N_P$ in $S$, and let $Q$ be a $N_O^q$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$ in $T$. Then $Nint(Ncl(Q))$ is a $N_{RO}^q$-nhd $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. By Corollary 3.3*[9]*, $Nint(Ncl(Q))$ is an $N_{\delta-OS}$ in $T$. By the hypothesis, $k^{-1}(Nint(Ncl(Q)))$ is a $N_{OS}$ in $S$. Since $Nint(Ncl(Q))\, qk\left(s_{(,\upsilon,\omega,\xi)}\right)$, we have $s_{(,\upsilon,\omega,\xi)} q k^{-1}(Nint(Ncl(Q)))$. Thus $s_{(,\upsilon,\omega,\xi)}$ does not belong to the set $\left(k^{-1}(Nint(Ncl(Q)))\right)^c$. Put $B = \left(k^{-1}(Nint(Ncl(Q)))\right)^c$. Since $B$ is a $N_{CS}$ and $s_{(,\upsilon,\omega,\xi)} \notin B = Ncl(B)$, there exists a $N_O^q$-nhd $P$ of $s_{(,\upsilon,\omega,\xi)}$ such that $P\tilde{q}B$. Then $s_{(,\upsilon,\omega,\xi)} qP \subseteq B^c = k^{-1}(Nint(Ncl(Q)))$. Thus $k(P) \subseteq Nint(Ncl(Q))$. Hence, $k$ is a $N_{ACF}$*

**Theorem 4.4.** *For the function $k : (S,\acute{\tau}) \to (T,\acute{\sigma})$ the following statements are equivalent:*

*(1) $k$ is a $N_{ACF}$.*
*(2) $Ncl\left(k^{-1}(Q)\right) \subseteq k^{-1}(Ncl_\delta(Q))$*
*(3) $Nint_\delta\left(k^{-1}(Q)\right) \subseteq k^{-1}(Nint(Q))$*

*Proof.*

- $(1) \Rightarrow (2)$**:** *Let $Q$ be a $N_S$ in $T$. Then $k^{-1}(Q)$ is a $N_S$ in $S$. By Theorem 4.3, $k\left(Ncl\left(k^{-1}(Q)\right)\right) \subseteq Ncl_\delta\left(k\left(k^{-1}(Q)\right)\right) \subseteq Ncl_\delta(Q)$. Thus $Ncl\left(k^{-1}(Q)\right) \subseteq k^{-1}(Ncl_\delta(Q))$.*
- $(2) \Rightarrow (1)$**:** *Let $P$ be a $N_S$ in $S$. Then $k(P)$ is a $N_S$ in $T$. By the hypothesis, $Ncl\left(k^{-1}(k(P))\right) \subseteq k^{-1}(Ncl_\delta(k(P)))$. Then $Ncl(P) \subseteq Ncl\left(k^{-1}(k(P))\right) \subseteq k^{-1}(Ncl_\delta(k(P)))$. Thus $Ncl(P) \subseteq Ncl_\delta(k(P))$. By Theorem 4.3, $k$ is a $N_{ACF}$.*
- $(2) \Rightarrow (3)$**:** *Let $Q$ be a $N_S$ in $T$. Then $Q^c$ is a $N_S$ in $T$. By the hypothesis, $Ncl\left(k^{-1}(Q^c)\right) \subseteq k^{-1}(Ncl_\delta(Q^c))$. Thus $k^{-1}(Nint_\delta(Q)) = k^{-1}\left((Ncl_\delta(Q^c))^c\right) =$
  $\left(k^{-1}(Ncl_\delta((Q^c)))\right)^c \subseteq \left(Ncl\left(k^{-1}(Q^c)\right)\right)^c = \left(Ncl\left(\left(k^{-1}(Q)\right)^c\right)\right)^c$
  $= Nint\left(k^{-1}(Q)\right)$.*
- $(3) \Rightarrow (2)$**:** *Let $Q$ be a $N_S$ in $T$. Then $Q^c$ is a $N_S$ in $T$. By the hypothesis, $k^{-1}(Nint_\delta(Q^c)) \subseteq Nint\left(k^{-1}(Q^c)\right)$. Thus $Ncl\left(k^{-1}(Q)\right) = \left(Nint\left(\left(k^{-1}(Q)\right)^c\right)\right)^c =$
  $\left(Nint\left(k^{-1}\left(Q^{-1}\right)\right)\right)^c \subseteq \left(k^{-1}(Nint\delta(Q^c))\right)^c = k^{-1}\left((Nint\delta(Q^c))^c\right) = k^{-1}(Nint_\delta(Q))$.
  Hence $Ncl\left(k^{-1}(Q)\right) \subseteq k^{-1}(Nint_\delta(Q))$*

**Corollary 4.5.** A function $k : (S,\acute{\tau}) \to (T,\acute{\sigma})$ is $N_{ACF}$ iff for each $N_P$ $s_{(,\upsilon,\omega,\xi)}$ in S and each $N_\delta$-nhd N of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$, the $N_S$ $k^{-1}(N)$ is a neutrosophic q(briefly $N^q$)-nhd of $s_{(,\upsilon,\omega,\xi)}$.

*proof.* Let $s_{(,\upsilon,\omega,\xi)}$ be a $N_P$ in S, and let N be an $N_\delta$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. Then there exists a $N^q_{RO}$-nhd Q of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$ such that $Q \subseteq N$. Since $k$ is a $N_{ACF}$, there exists a $N^q_O$-nhd P of $s_{(,\upsilon,\omega,\xi)}$ such that $k(P) \subseteq Nint(Ncl(Q)) = Q \subseteq N$. Thus, there exists a $N_{OS}$ P such that $s_{(,\upsilon,\omega,\xi)}qP \subseteq k^{-1}(N)$. Hence $k^{-1}(N)$, is a $N^q$-nhd of $s_{(,\upsilon,\omega,\xi)}$.
Conversely, let $s_{(,\upsilon,\omega,\xi)}$ be a $N_P$ in S, and let Q be a $N^q$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. Then, $Nint(Ncl(Q))$ is a $N^q_{RO}$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. $Nint(Ncl(Q))$ is a $N_\delta$-nhd of $k\left(s_{(,\upsilon,\omega,\xi)}\right)$. By the hypothesis, $k^{-1}(Nint(Ncl(Q)))$ is a $N^q$-nhd of $s_{(,\upsilon,\omega,\xi)}$. Since, $k^{-1}(Nint(Ncl(Q)))$ is a $N^q$-nhd of $s_{(,\upsilon,\omega,\xi)}$, there exists a $N^q_O$-nhd P of $s_{(,\upsilon,\omega,\xi)}$ such that $P \subseteq k^{-1}(Nint(Ncl(Q)))$. Thus, there exists a $N^q_O$-nhd P of $s_{(,\upsilon,\omega,\xi)}$ such that $k(P) \subseteq Nint(Ncl(Q))$. Hence, $k$ is a $N_{ACF}$.

## 5. Conclusion and future work

The proposed research work defines $N_{\delta-CF}$ and $N_{ACF}$ in $N_{TS}$. The properties of $N_{\delta-CF}$ and $N_{ACF}$ were discussed in detail. Further, $N_{\delta-CF}$and $N_{ACF}$ can be derived into functions such as $N_\delta$-strongly continuous and $N_\delta$-perfectly continuous functions in $N_{TS}$. In addition, to that, it can be extended to $N_\delta$-irresolute functions in $N_{TS}$.

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