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DECOMPOSITION OF δ-CONTINUITY VIA e-OPEN SET

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Abstract. The main purpose of this paper is to obtain a new decomposition of δ-continuity via e-open set which is defined by Ekici [7]. For this aim, we introduce the notion of δ-locally e-closed set which is weaker than the notion of locally δ-closed set. Also, we investigate many fundamental properties of it. Finally, we obtain two new decompositions of the notion of δ-continuity.

1. Introduction

The notion of decomposition of continuity is one of the most important notions in general topology. Recently the notion of decomposition have been studied many mathematicians such as Tong [18], Ganster and Reilly [8], Przemski [15], Hatice [10], Al-Nashef [1], Hatice and Noiri [11], Noiri and Sayed [12], Erguang and Pengfei [4]. Furthermore, the concepts of t-set and B-set in topological spaces were defined and studied by Tong [18] in 1989. They have obtained different decompositions of continuity on their studies via these concepts.

In this paper, we define and study the notions of δ-locally-e-closed sets, δ-e-t-set, δ-e-B-set, δ-locally e-closed continuous function via the concept of e-open set defined by Ekici [7], and obtain decomposition of δ-continuity.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let X be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively. The family of all closed (open) sets of X is denoted by $C(X,\tau)(O(X))$. Recall that a set A is called regular open [17] (resp. regular closed [17]) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A subset A of a space (X, τ) is called δ-open [19] if for each $x \in A$ there exists a regular open set V

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such that $x \in V \subseteq A$. A set A is said to be δ-closed if its complement is δ-open. The intersection of all regular closed sets of X containing A is called the δ-closure [19] of A and is denoted by $\delta\text{-}cl(A)$. The union of all regular open sets of X contained in A is called the δ-interior [19] of A and is denoted by $\delta\text{-}int(A)$.

A subset A of a space X is called e-open [7] (resp. a-open [6], δ-preopen [16], δ-semiopen [14]) if $A \subseteq cl(\delta\text{-}int(A)) \cup int(\delta\text{-}cl(A))$ (resp. $A \subseteq int(cl(\delta\text{-}int(A)))$, $A \subseteq int(\delta\text{-}cl(A))$, $A \subseteq cl(\delta\text{-}int(A))$. The complement of an e-open (resp. a-open, δ-preopen, δ-semiopen) set is called e-closed [7] (resp. a-closed [6], δ-preclosed [16], δ-semiclosed [14]). The intersection of all e-closed (resp. δ-preclosed, δ-semiclosed) sets of X containing A is called the e-closure [7] (resp. δ-preclosure [16], δ-semiclosure [14]) of A and is denoted by $e\text{-}cl(A)$ (resp. $\delta\text{-}pcl(A)$, $\delta\text{-}scl(A)$). The union of all e-open (resp. δ-preopen, δ-semiopen) sets of X contained in A is called the e-interior [7](resp. δ-preinterior [16], δ-semiinterior [14]) of A and is denoted by $e\text{-}int(A)$ (resp. $\delta\text{-}pint(A)$, $\delta\text{-}sint(A)$).

The family of all e-open (resp. e-closed, a-open, δ-preopen, δ-semiopen) subsets of X is denoted by $eO(X)$ (resp. $eC(X)$, $aO(X)$, $\delta PO(X)$, $\delta SO(X)$). The family of all e-open (resp. e-closed, a-open, δ-preopen, δ-semiopen) sets of X containing a point x of X is denoted by $eO(X,x)$ (resp. $eC(X,x)$, $aO(X,x)$, $\delta PO(X,x)$, $\delta SO(X,x)$).

Lemma 2.1. [7] *Let A be a subset of a space X. A is e-open if and only if $A = e\text{-}int(A)$.*

3. δ-locally e-closed sets

Definition 3.1. A subset A of a space X is called:
(1) δ-t-set if $\delta\text{-}int(A) = \delta\text{-}int(\delta\text{-}cl(A))$.
(2) δ-B-set if $A = U \cap V$, where $U \in \delta O(X)$ and V is δ-t-set.
(3) locally closed [3] if $A = U \cap V$, where $U \in O(X)$ and $V \in C(X)$.
(4) locally δ-closed [13] if $A = U \cap V$, where $U \in \delta O(X)$ and $V \in \delta C(X)$.
(5) δ-locally e-closed if $A = U \cap V$, where $U \in \delta O(X)$ and $V \in eC(X)$.

The collection of all locally δ-closed sets (resp. δ-locally e-closed sets) in X is denoted by $L\delta C(X)$ (resp. $\delta LeC(X)$).

Definition 3.2. [3] A space X is called extremally disconnected if the closure of every open set of X is open in X.

Theorem 3.3. *Let A be a subset of a topological space X. Then A is locally δ-closed if and only if $A = U \cap \delta\text{-}cl(A)$ for some δ-open set U.*

Proof. Necessity. Let $A \in L\delta C(X)$.
$A \in L\delta C(X) \Rightarrow (\exists U \in \delta O(X))(\exists V \in \delta C(X))(A = U \cap V)$
$\Rightarrow (\exists U \in \delta O(X))(A \subseteq \delta\text{-}cl(A) = \delta\text{-}cl(U \cap V) \subseteq \delta\text{-}cl(U) \cap \delta\text{-}cl(V) = \delta\text{-}cl(U) \cap V)$
$\Rightarrow (\exists U \in \delta O(X))(A = U \cap A \subseteq U \cap \delta\text{-}cl(A) \subseteq U \cap (\delta\text{-}cl(U) \cap V) = U \cap V = A)$
$\Rightarrow (\exists U \in \delta O(X))(A = U \cap \delta\text{-}cl(A))$.
Sufficiency. Let $A \subseteq X$.
$$\left.\begin{array}{r} V := \delta\text{-}cl(A) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow (\exists U \in \delta O(X))(\exists V \in \delta C(X))(A = U \cap V) \Rightarrow A \in L\delta C(X). \qquad \square$$

Remark 3.4. The notions of e-open set and locally δ-closed set are independent as shown by the following example.

Example 3.5. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then $eO(X) = 2^X \setminus \{\{d\}, \{a, d\}, \{c, d\}\}$ and $L\delta C(X) = \{\emptyset, X, \{b\}, \{d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}\}$. It is clear that $\{c\} \in eO(X)$ but $\{c\} \notin L\delta C(X)$ and $\{d\} \in L\delta C(X)$ but $\{d\} \notin eO(X)$.

Lemma 3.6. *Let (X, τ) be an extremally disconnected space. Then $cl(\delta\text{-}int(A)) \subseteq int(\delta\text{-}cl(A))$ for any subset of a space X.*

Proof. Let A be any subset of X.

$$\left.\begin{array}{r} A \subseteq X \Rightarrow \delta\text{-}int(A) \subseteq A \Rightarrow int(cl(\delta\text{-}int(A))) \subseteq int(cl(A)) \\ (X, \tau) \text{ is extremally disconnected} \end{array}\right\} \Rightarrow$$

$$\Rightarrow cl(\delta\text{-}int(A)) = int(cl(\delta\text{-}int(A))) \subseteq int(cl(A)) \subseteq int(\delta\text{-}cl(A)).$$ □

Theorem 3.7. *Let A be a subset of X and (X, τ) be an extremally disconnected space. The following are equivalent:*
(1) A is δ-open,
(2) A is e-open and locally δ-closed.

Proof. (1) $\Rightarrow$ (2) : Obvious.
(2) $\Rightarrow$ (1) : Let $A \in eO(X) \cap L\delta C(X)$.

$$\begin{array}{lcl} A \in eO(X) \cap L\delta C(X) \Rightarrow (A \in eO(X))(A \in L\delta C(X)) & & \\ \Rightarrow (A \subseteq int(\delta\text{-}cl(A)) \cup cl(\delta\text{-}int(A)))(\exists U \in \delta O(X))(A = U \cap \delta\text{-}cl(A)) & & \\ \Rightarrow A = U \cap A & \subseteq & U \cap [int(\delta\text{-}cl(A)) \cup cl(\delta\text{-}int(A))] \\ & = & [U \cap int(\delta\text{-}cl(A))] \cup [U \cap cl(\delta\text{-}int(A))] \\ & = & [U \cap \delta\text{-}int(\delta\text{-}cl(A))] \cup [U \cap cl(\delta\text{-}int(A))] \\ & \overset{\text{Lemma 3.6.}}{\subseteq} & [\delta\text{-}int(U \cap \delta\text{-}cl(A))] \cup [U \cap int(\delta\text{-}cl(A))] \\ & = & [\delta\text{-}int(U \cap \delta\text{-}cl(A))] \cup [\delta\text{-}int(U \cap \delta\text{-}cl(A))] \\ & = & \delta\text{-}int(A) \cup \delta\text{-}int(A) = \delta\text{-}int(A) \end{array}$$

Then we have $A = \delta\text{-}int(A)$. Therefore $A \in \delta O(X)$. □

Definition 3.8. A subset A of a topological space X is called $D(\delta\text{-}c, e)$-set if $\delta\text{-}int(A) = e\text{-}int(A)$.

Remark 3.9. The notions of e-open set and $D(\delta\text{-}c, e)$-set are independent as shown by the following example.

Example 3.10. Let $X := \{a, b, c\}$ and $\tau := \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then $eO(X) = 2^X \setminus \{\{c\}\}$. It is clear that $\{a, c\} \in eO(X)$ but $\{a, c\}$ is not $D(\delta\text{-}c, e)$-set and $\{c\}$ is $D(\delta\text{-}c, e)$-set but $\{c\} \notin eO(X)$.

Theorem 3.11. *Let A be a subset of a topological space X. The following are equivalent:*
(1) A is δ-open,
(2) A is e-open and $D(\delta\text{-}c, e)$-set.

Proof. (1) $\Rightarrow$ (2) : Let $A \in \delta O(X)$.

$$\begin{array}{rcl} A \in \delta O(X) & \Rightarrow & (A = \delta\text{-}int(A))(A \in eO(X)) \\ & \Rightarrow & (A = \delta\text{-}int(A))(A = e\text{-}int(A)) \\ & \Rightarrow & \delta\text{-}int(A) = e\text{-}int(A) \\ & \Rightarrow & A \text{ is } D(\delta\text{-}c, e)\text{-set}. \end{array}$$

$(2) \Rightarrow (1)$: Let A be e-open and $D(\delta\text{-}c, e)$-set.
$(A \in eO(X))(A \text{ is } D(\delta\text{-}c, e)\text{-set}) \Rightarrow A = e\text{-}int(A) = \delta\text{-}int(A) \Rightarrow A \in \delta O(X)$. □

Theorem 3.12. *Let A be a subset of a topological space X. Then A is δ-locally e-closed if and only if there exists a δ-open U of X such that $A = U \cap e\text{-}cl(A)$.*

Proof. Necessity. Let $A \in \delta LeC(X)$.
$A \in \delta LeC(X) \Rightarrow (\exists U \in \delta O(X))(\exists V \in eC(X))(A = U \cap V)$
$\Rightarrow (\exists U \in \delta O(X))(A \subseteq e\text{-}cl(A) = e\text{-}cl(U \cap V) \subseteq e\text{-}cl(U) \cap e\text{-}cl(V) = e\text{-}cl(U) \cap V)$
$\Rightarrow (\exists U \in \delta O(X))(A = U \cap A \subseteq U \cap e\text{-}cl(A) \subseteq U \cap (e\text{-}cl(U) \cap V) = U \cap V = A)$
$\Rightarrow (\exists U \in \delta O(X))(A = U \cap e\text{-}cl(A))$.
Sufficiency. Obvious. □

Theorem 3.13. *Let A be a subset of a topological space X if A is δ-locally e-closed, then the following hold.*
(1) $e\text{-}cl(A) \setminus A$ is e-closed.
(2) $A \cup (X \setminus e\text{-}cl(A))$ is e-open.
(3) $A \subseteq e\text{-}int(A \cup (X \setminus e\text{-}cl(A)))$.

Proof. (1) : Let $A \in \delta LeC(X)$.
$A \in \delta LeC(X) \Rightarrow (\exists U \in \delta O(X))(A = U \cap e\text{-}cl(A))$
$\Rightarrow (U \in \delta O(X))(e\text{-}cl(A) \setminus A = e\text{-}cl(A) \setminus (U \cap e\text{-}cl(A)))$
$\Rightarrow (U \in \delta O(X))(e\text{-}cl(A) \setminus A = e\text{-}cl(A) \cap [X \setminus (U \cap e\text{-}cl(A))])$
$\Rightarrow (U \in \delta O(X))(e\text{-}cl(A) \setminus A = e\text{-}cl(A) \cap [(X \setminus U) \cup (X \setminus e\text{-}cl(A))])$
$\Rightarrow (U \in \delta O(X))(e\text{-}cl(A) \setminus A = [e\text{-}cl(A) \cap (X \setminus U)] \cup [e\text{-}cl(A) \cap (X \setminus e\text{-}cl(A))])$
$\Rightarrow (U \in \delta O(X))(e\text{-}cl(A) \setminus A = e\text{-}cl(A) \cap (X \setminus U) \in eC(X))$
(2) : Let $A \in \delta LeC(X)$.

$$\begin{array}{lcl} A \in \delta LeC(X) & \overset{(1)}{\Rightarrow} & e\text{-}cl(A) \setminus A \in eC(X) \\ & \Rightarrow & X \setminus (e\text{-}cl(A) \setminus A) \in eO(X) \\ & \Rightarrow & X \setminus (e\text{-}cl(A) \cap (X \setminus A)) \in eO(X) \\ & \Rightarrow & A \cup (X \setminus e\text{-}cl(A)) \in eO(X). \end{array}$$

(3) : Let $A \in \delta LeC(X)$.

$$\begin{array}{lcl} A \in \delta LeC(X) & \overset{(2)}{\Rightarrow} & A \cup (X \setminus e\text{-}cl(A)) \in eO(X) \\ & \overset{\text{Lemma 2.1.}}{\Rightarrow} & A \subseteq A \cup (X \setminus e\text{-}cl(A)) = e\text{-}int(A \cup (X \setminus e\text{-}cl(A))). \end{array}$$ □

Proposition 3.14. Let A be a subset of a topological space X. Then A is δ-open if and only if A is δ-preopen set and δ-B-set.

Proof. Necessity. Obvious.
Sufficiency. Let $A \in \delta PO(X)$ and A is δ-B-set.

$$\left.\begin{array}{r} A \in \delta PO(X) \Rightarrow A \subseteq int(\delta\text{-}cl(A)) \\ A \text{ is } \delta\text{-}B\text{-}set \Rightarrow (\exists U \in \delta O(X))(\exists V \text{ is } \delta\text{-t-set})(A = U \cap V) \end{array}\right\} \Rightarrow$$

$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-t-set})(A \subseteq int(\delta\text{-}cl(U \cap V)))$
$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-t-set})(A \subseteq int(\delta\text{-}cl(U \cap V)) \subseteq int[\delta\text{-}cl(U) \cap \delta\text{-}cl(V)])$
$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-t-set})(A \subseteq int(\delta\text{-}cl(U)) \cap int(\delta\text{-}cl(V)) = int(\delta\text{-}cl(U)) \cap \delta\text{-}int(V))$
$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-t-set})(U \cap A = A \subseteq U \cap [int(\delta\text{-}cl(U)) \cap \delta\text{-}int(V)] = U \cap \delta\text{-}int(V))$
$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-t-set})(A \subseteq \delta\text{-}int(U \cap V) = \delta\text{-}int(A))$
Then we have $A = \delta\text{-}int(A)$. Therefore $A \in \delta O(X)$. □

Proposition 3.15. [5] Let A be a subset of a topological space X. Then A is a-open if and only if A is δ-preopen set and δ-semiopen set.

Lemma 3.16. *Let X be a topological space. If $A \in \delta SO(X)$, then δ-$pcl(A) = \delta$-$cl(A)$.*

Definition 3.17. A subset A of a space X is called δ-locally δ-semiclosed if $A = U \cap V$, where $U \in \delta O(X)$ and V is δ-semiclosed. The collection of all δ-locally δ-semiclosed sets in X will be denoted by $\delta L\delta SC(X)$. It is easy to see that every δ-locally δ-semiclosed set is δ-B-set.

Proposition 3.18. Let A be a subset of a topological space X. Then A is δ-locally δ-semiclosed if and only if $A = U \cap \delta$-$scl(A)$ for some δ-open set U.

Proof. It is similar to Theorem 3.3. □

Lemma 3.19. *A subset A in a topological space X is δ-B-set if it is δ-locally e-closed and δ-semiopen.*

Proof. Let $A \in \delta LeC(X)$ and $A \in \delta SO(X)$.
$A \in \delta LeC(X) \cap \delta SO(X) \Rightarrow (A \in \delta LeC(X))(A \in \delta SO(X))$
$\Rightarrow (\exists U \in \delta O(X))(A = U \cap e\text{-}cl(A))(A \in \delta SO(X))$
$\Rightarrow (\exists U \in \delta O(X))(A = U \cap (\delta\text{-}scl(A) \cap \delta\text{-}pcl(A)))(A \in \delta SO(X))$
$\overset{\text{Lemma 3.16}}{\Rightarrow} (\exists U \in \delta O(X))(A = U \cap (\delta\text{-}scl(A) \cap \delta\text{-}cl(A)))$
$\Rightarrow (\exists U \in \delta O(X))(A = U \cap \delta\text{-}scl(A))$
$\Rightarrow A \in \delta L\delta SC(X)$
Since $A \in \delta L\delta SC(X)$, so A is δ-B-set. □

Remark 3.20. The converse of Lemma 3.19 is not true in general as shown by the following example.

Example 3.21. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. It is clear that $\{d\}$ is δ-B-set but $\{d\} \notin \delta SO(X)$.

Remark 3.22. The notions of a-open sets and δ-locally e-closed sets are independent as shown by the following examples.

Example 3.23. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Then $aO(X) = O(X)$ and $\delta LeC(X) = 2^X \backslash \{\{a, c, d\}, \{b, c, d\}\}$. It is clear that $\{a, c, d\} \in aO(X)$ but $\{a, c, d\} \notin \delta LeC(X)$ and $\{a, b, c\} \in \delta LeC(X)$ but $\{a, b, c\} \notin aO(X)$.

Theorem 3.24. *Let A be a subset of a topological space X. The following are equivalent:*
(1) A is δ-open,
(2) A is a-open set and δ-locally e-closed.

Proof. It is clear from Proposition 3.14, Proposition 3.15 and Lemma 3.19. □

Definition 3.25. [2] A subset A of a space X is called generalized e-closed (briefly ge-closed) if e-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. The complement of generalized e-closed is called generalized e-open (briefly ge-open). The family of all ge-closed (resp. ge-open) subsets of X will be denoted by $geC(X)$(resp. $geO(X)$).

Theorem 3.26. *Let A be a subset of a topological space X. The following are equivalent:*
(1) A is e-closed,
(2) A is ge-closed and δ-locally e-closed.

Proof. (1) $\Rightarrow$ (2) : Obvious.
(2) $\Rightarrow$ (1) : Let $A \in \delta LeC(X) \cap geC(X)$.

$$\begin{aligned}
&A \in \delta LeC(X) \cap geC(X) \Rightarrow (A \in \delta LeC(X))(A \in geC(X)) \\
&\left.\begin{aligned}\Rightarrow (\exists U \in \delta O(X))(A = U \cap e\text{-}cl(A))(A \in geC(X)) \\ \delta O(X) \subseteq O(X)\end{aligned}\right\} \Rightarrow \\
&\Rightarrow (\exists U \in O(X))(A \subseteq U)(A \in geC(X)) \\
&\Rightarrow e\text{-}cl(A) \subseteq U \\
&\Rightarrow e\text{-}cl(A) \subseteq U \cap e\text{-}cl(A) = A
\end{aligned}$$

Then we have $A = e\text{-}cl(A)$. Therefore $A \in eC(X)$. □

4. δ-e-t-sets

Definition 4.1. A subset A of a space X is said to be:
(1) δ-e-t-set if $\delta\text{-}int(A) = \delta\text{-}int(e\text{-}cl(A))$,
(2) δ-e-B-set if $A = U \cap V$, where $U \in \delta O(X)$ and V is δ-e-t-set,
(3) δ-e-semiopen if $A \subseteq \delta\text{-}cl(e\text{-}int(A))$,
(4) δ-e-preopen if $A \subseteq \delta\text{-}int(e\text{-}cl(A))$.

Proposition 4.2. For subsets A and B of a space X, the following properties hold:
(1) A is δ-e-t-set if and only if it is δ-e-semiclosed,
(2) If A is e-closed, then it is δ-e-t-set,
(3) If A and B are δ-e-t-set, then $A \cap B$ is δ-e-t-set.

Proof. (1) : *Necessity.* Let A be δ-e-t-set.
A is δ-e-t-set $\Rightarrow \delta\text{-}int(A) = \delta\text{-}int(e\text{-}cl(A)) \subseteq A \Rightarrow A$ is δ-e-semiclosed.
Sufficiency. Let A be δ-e-semiclosed.

$$\begin{aligned}
A \text{ is } \delta\text{-}e\text{-semiclosed} \quad &\Rightarrow \quad \delta\text{-}int(e\text{-}cl(A)) \subseteq A \\
&\Rightarrow \quad \delta\text{-}int(e\text{-}cl(A)) \subseteq \delta\text{-}int(A) \\
&\Rightarrow \quad \delta\text{-}int(e\text{-}cl(A)) = \delta\text{-}int(A)
\end{aligned}$$

Then A is δ-e-t-set.
(2) : Obvious.
(3) : Let A and B be δ-e-t-sets.

$$\begin{aligned}
\delta\text{-}int(A \cap B) \quad &\subseteq \quad \delta\text{-}int(e\text{-}cl(A \cap B)) \\
&\subseteq \quad \delta\text{-}int[e\text{-}cl(A) \cap e\text{-}cl(B)] \\
&= \quad \delta\text{-}int[e\text{-}cl(A)] \cap \delta\text{-}int[e\text{-}cl(B)] \\
&\overset{\text{Hypothesis}}{=} \quad \delta\text{-}int(A) \cap \delta\text{-}int(B) \\
&= \quad \delta\text{-}int(A \cap B)
\end{aligned}$$

Then we have $\delta\text{-}int(A \cap B) = \delta\text{-}int(e\text{-}cl(A \cap B))$. Therefore $A \cap B$ is δ-e-t-set. □

Remark 4.3. The converse of Proposition 4.2.(2) is not true in general. Namely, δ-e-t-set need not be e-closed.

Example 4.4. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then $eC(X) = 2^X \setminus \{\{a, b\}, \{b, c\}, \{a, b, c\}\}$. It is clear that $\{b, c\}$ is δ-e-t-set but $\{b, c\} \notin eC(X)$.

Proposition 4.5. For a subset A of a space X, the following properties hold:
(1) If A is δ-t-set, then it is δ-e-t-set.

(2) If A is δ-e-t-set, then it is δ-e-B-set.
(3) If A is δ-B-set, then it is δ-e-B-set.

Proof. (1): Let A be δ-t-set.

$$\begin{array}{rll} A \text{ is a } \delta\text{-}t\text{-set} & \Rightarrow & \delta\text{-}int(A) = \delta\text{-}int(\delta\text{-}cl(A)) \\ & \Rightarrow & \delta\text{-}int(A) \subseteq \delta\text{-}int(e\text{-}cl(A)) \subseteq \delta\text{-}int(\delta\text{-}cl(A)) = \delta\text{-}int(A) \\ & \Rightarrow & \delta\text{-}int(A) = \delta\text{-}int(e\text{-}cl(A)) \\ & \Rightarrow & A \text{ is } \delta\text{-}e\text{-}t\text{-set.} \end{array}$$

(2) and (3) : Obvious. □

Theorem 4.6. *Let A be a subset of a topological space X. The following are equivalent:*
(1) A is δ-open,
(2) A is δ-e-preopen and δ-e-B-set.

Proof. (1) $\Rightarrow$ (2) : Let A be δ-open.

$$\left.\begin{array}{r} A \in \delta O(X) \Rightarrow A = \delta\text{-}int(A) \subseteq \delta\text{-}int(e\text{-}cl(A)) \Rightarrow A \text{ is } \delta\text{-}e\text{-preopen} \\ (U := A)(V := X) \end{array}\right\} \Rightarrow$$

$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-}e\text{-}t\text{-set})(A = U \cap V)$
$\Rightarrow A$ is δ-e-B-set.

(2) $\Rightarrow$ (1) : Let A is e-preopen and e-B-set.

$$\left.\begin{array}{r} A \text{ is } \delta\text{-}e\text{-}B\text{-set} \Rightarrow (\exists U \in \delta O(X))(\exists V \text{ is } \delta\text{-}e\text{-}t\text{-set})(A = U \cap V) \\ A \text{ is } e\text{-preopen} \Rightarrow A \subseteq \delta\text{-}int(e\text{-}cl(A)) \end{array}\right\} \Rightarrow$$

$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-}e\text{-}t\text{-set})(A \subseteq \delta\text{-}int(e\text{-}cl(U \cap V)))$
$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-}e\text{-}t\text{-set})(A \subseteq \delta\text{-}int(e\text{-}cl(U \cap V)) \subseteq \delta\text{-}int[e\text{-}cl(U) \cap e\text{-}cl(V)])$
$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-}e\text{-}t\text{-set})(A \subseteq \delta\text{-}int(e\text{-}cl(U)) \cap \delta\text{-}int(e\text{-}cl(V)) = \delta\text{-}int(e\text{-}cl(U)) \cap \delta\text{-}int(V))$
$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-}e\text{-}t\text{-set})(U \cap A = A \subseteq U \cap [\delta\text{-}int(e\text{-}cl(U)) \cap \delta\text{-}int(V)] = U \cap \delta\text{-}int(V))$
$\Rightarrow (U \in \delta O(X))(V \text{ is } \delta\text{-}e\text{-}t\text{-set})(A \subseteq \delta\text{-}int(U \cap V) = \delta\text{-}int(A))$

Then we have $A = \delta\text{-}int(A)$. Therefore $A \in \delta O(X)$. □

Remark 4.7. The notions of δ-e-preopen sets and δ-e-B-sets are independent as shown by the following examples.

Example 4.8. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Then $eC(X) = 2^X \setminus \{\{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clear that $\{b, c\}$ is δ-e-B-set but $\{b, c\}$ is not δ-e-preopen. And it is clear that $\{a, c, d\}$ is δ-e-preopen but $\{a, c, d\}$ is not δ-e-B-set.

5. Decompositions of δ-continuity

Definition 5.1. A function $f : X \to Y$ is called e-continuous [7] (resp. a-continuous [6], δ-semi continuous, δ-B-continuous, locally δ-closed continuous [13], $D(\delta\text{-}c, e)$-continuous, δ-locally e-closed continuous) if $f^{-1}[V]$ is e-open (resp. a-open, δ-semiopen, δ-B-set, locally δ-closed, $D(\delta\text{-}c, e)$-set, δ-locally e-closed) in X for each open set V of Y.

Theorem 5.2. *Let $f : X \to Y$ be a function. Then*
(1) If X is extremally disconnected, f is δ-continuous if and only if f is e-continuous and locally δ-closed continuous.
(2) f is δ-continuous if and only if f is e-continuous and $D(\delta\text{-}c, e)$-continuous.
(3) f is δ-continuous if and only if f is a-continuous and δ-locally e-closed continuous.

Proof. (1): It is clear from Theorem 3.7.
(2): It is clear from Theorem 3.11.
(3): It is clear from Theorem 3.24. □

Remark 5.3. The notions of e-continuous functions and $D(\delta\text{-}c, e)$-continuous functions are independent as shown by the following examples.

Example 5.4. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define the function $f : (X, \tau) \to (X, \tau)$ by $f := \{(a, c), (b, c), (c, d), (d, d)\}$. Then f is $D(\delta\text{-}c, e)$-continuous, but it is not e-continuous. And a function $f : (X, \tau) \to (X, \tau)$ such that $f := \{(a, b), (b, c), (c, c), (d, a)\}$. Then f is e-continuous, but it is not $D(\delta\text{-}c, e)$-continuous.

Remark 5.5. The notions of e-continuous functions and locally δ-closed continuous functions are independent as shown by the following examples.

Example 5.6. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Define the function $f : (X, \tau) \to (X, \tau)$ by $f := \{(a, a), (b, c), (c, a), (d, b)\}$. Then f is locally δ-closed continuous, but it is not e-continuous. And a function $f : (X, \tau) \to (X, \tau)$ such that $f := \{(a, d), (b, a), (c, b), (d, a)\}$. Then f is e-continuous, but it is not locally δ-closed continuous.

Remark 5.7. The notions of a-continuous functions and δ-locally e-closed continuous functions are independent as shown by the following examples.

Example 5.8. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define the function $f : (X, \tau) \to (X, \tau)$ by $f := \{(a, a), (b, a), (c, b), (d, c)\}$. Then f is δ-locally e-closed continuous, but it is not a-continuous. And a function $f : (X, \tau) \to (X, \tau)$ such that $f := \{(a, b), (b, a), (c, c), (d, c)\}$. Then f is a-continuous, but it is not δ-locally e-closed continuous.

Definition 5.9. A function $f : X \to Y$ is called:
1) contra e-continuous [9] if $f^{-1}[V]$ is e-closed in X for each open set V of Y.
2) contra ge-continuous if $f^{-1}[V]$ is ge-closed in X for each open set V of Y.

Theorem 5.10. *Let $f : X \to Y$ be a function. Then f is contra e-continuous if and only if it is δ-locally e-closed continuous and contra ge-continuous.*

Proof. It is clear from Theorem 3.26. □

Theorem 5.11. *Let $f : X \to Y$ be function. f is δ-B-continuous if it is δ-locally e-closed continuous and δ-semi continuous.*

Proof. It is clear from Lemma 3.19. □

Definition 5.12. A function $f : X \to Y$ is called:
1) δ-e-pre-continuous if $f^{-1}[V]$ is δ-e-preopen in X for each open set V of Y.
2) δ-e-B-continuous if $f^{-1}[V]$ is δ-e-B-set in X for each open set V of Y.

Theorem 5.13. *Let $f : X \to Y$ be a function. Then f is δ-continuous if and only if it is δ-e-pre-continuous and δ-e-B-continuous.*

Proof. It is clear from Theorem 4.6. □

Remark 5.14. The notions of δ-e-pre-continuous functions and δ-e-B-continuous functions are independent as shown by the following examples.

Example 5.15. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Define the function $f : (X, \tau) \to (X, \tau)$ by $f := \{(a, a), (b, d), (c, a), (d, b)\}$. Then f is δ-e-B-continuous, but it is not δ-e-pre-continuous.

Example 5.16. Let $X := \{a, b, c, d\}$ and $\tau := \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define the function $f : (X, \tau) \to (X, \tau)$ by $f := \{(a, b), (b, a), (c, c), (d, c)\}$. Then f is δ-e-pre-continuous, but it is not δ-e-B-continuous.

6. Conclusion

The notion of decomposition of continuity is one of the most important subject in general topology. In order to obtain some decompositions of δ-continuity, we defined and studied the concepts of δ-locally-e-closed set, δ-e-B-set and $D(\delta\text{-}c, e)$-set. We gave many examples related to these concepts. Moreover, we examined the relationships between these concepts and the concept of δ-open sets and found many results. Based on these results, we obtained some decompositions of the δ-continuity. We believe that this study will help researchers to upgrade and support the further studies on decompositions of various continuities.

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