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ON $\mathcal{N}_{*g\alpha}$- CLOSED SET IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract. In this article, we introduce the conception of $\mathcal{N}_{*g\alpha}$-Closed Set(briefly $\mathcal{N}_{*g\alpha}$-CS) in neutrosophic topological spaces(briefly NTS). Further establishes their properties.

1. Introduction

The denotation of neutrosophic set was commenced by Smarandache[9] and clarified that the neutrosophic set is a generalization of intuitionistic fuzzy set (IFS). Salama and Alblowi[6] proposed the conception of neutrosophic topological space (NTS) in 2012 that had been investigated recently. All the elements within neutrosophic set have the degree of membership, indefiniteness and degree of non-membership values. Arokiarani et al.[2] introduced the α-closed set in NTS. Dhavaseelan and Saied Jafari[3] introduced neutrosophic generalized closed sets in 2017. Sreeja et al.[10] studied the denotation of neutrosophic gα-closed sets and neutrosophic gα-open sets in NTS. Vigneshwaran et al.[11] defined a new closed set as $^*g\alpha$-closed sets in topological spaces which has been applied to define some topological functions as continuous functions, irresolute functions and homeomorphic functions with some separable axioms. The purpose of this paper is to incorporate the idea of $\mathcal{N}_{*g\alpha}$-CS in NTS and also study their properties.

2. Preliminaries

In this section, we recall some of basic definitions which was already defined by various authors and useful in the sequel.

Definition 2.1. [8] Let $\mathcal{W}$ be a non empty fixed set. A neutrosophic set $\mathcal{S}$ is an object of the following form

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$$\mathcal{S} = \{\langle w, \mu_{\mathcal{S}}(w), \sigma_{\mathcal{S}}(w), \gamma_{\mathcal{S}}(w) : w \in \mathcal{W}\rangle\}$$

where $\mu_{\mathcal{S}}(w)$, $\sigma_{\mathcal{S}}(w)$ and $\gamma_{\mathcal{S}}(w)$ denote the degree of membership, the degree of indeterminacy and the degree of non membership for each element $w \in \mathcal{W}$ to the set $\mathcal{S}$, respectively.

Definition 2.2. [8] Let $\mathcal{S}$ and $\mathcal{R}$ be neutrosophic sets of the form
$\mathcal{S} = \{\langle w, \mu_{\mathcal{S}}(w), \sigma_{\mathcal{S}}(w), \gamma_{\mathcal{S}}(w) : w \in \mathcal{W}\rangle\}$ and
$\mathcal{R} = \{\langle w, \mu_{\mathcal{R}}(w), \sigma_{\mathcal{R}}(w), \gamma_{\mathcal{R}}(w) : w \in \mathcal{W}\rangle\}$. Then

- $\mathcal{S} \subseteq \mathcal{R}$ if and only if $\mu_{\mathcal{S}}(w) \leq \mu_{\mathcal{R}}(w)$, $\sigma_{\mathcal{S}}(w) \geq \sigma_{\mathcal{R}}(w)$ and $\gamma_{\mathcal{S}}(w) \geq \gamma_{\mathcal{R}}(w)$;
- $(\mathcal{S})^c$=$\{\langle w, \gamma_{\mathcal{S}}(w), 1 - \sigma_{\mathcal{S}}(w), \mu_{\mathcal{S}}(w) : w \in \mathcal{W}\rangle\}$;
- $\mathcal{S} \cup \mathcal{R}$=$\{\langle w, \mu_{\mathcal{S}}(w) \vee \mu_{\mathcal{R}}(w), \sigma_{\mathcal{S}}(w) \wedge \sigma_{\mathcal{R}}(w), \gamma_{\mathcal{S}}(w) \wedge \gamma_{\mathcal{R}}(w)\rangle : w \in \mathcal{W}\}$;
- $\mathcal{S} \cap \mathcal{R}$=$\{\langle w, \mu_{\mathcal{S}}(w) \wedge \mu_{\mathcal{R}}(w), \sigma_{\mathcal{S}}(w) \vee \sigma_{\mathcal{R}}(w), \gamma_{\mathcal{S}}(w) \vee \gamma_{\mathcal{R}}(w)\rangle : w \in \mathcal{W}\}$.

Definition 2.3. [8] $0_{\mathcal{N}}, 1_{\mathcal{N}}$ is of the form

- $0_{\mathcal{N}} = \{\langle w, (0, 1, 1) : w \in \mathcal{W}\rangle\}$
- $1_{\mathcal{N}} = \{\langle w, (1, 0, 0) : w \in \mathcal{W}\rangle\}$.

Definition 2.4. [6] A neutrosophic topology in a nonempty set $\mathcal{W}$ is a family ς of neutrosophic sets in $\mathcal{W}$ satisfying the following axioms:

- $0_{\mathcal{N}}, 1_{\mathcal{N}} \in \varsigma$
- $\mathcal{S} \cap \mathcal{R} \in \varsigma$ for any $\mathcal{S}, \mathcal{R} \in \varsigma$
- $\cup(\mathcal{S})_i$ for any arbitrary family $\{(\mathcal{S})_i : i \in J\} \subseteq \varsigma$.

Definition 2.5. [6] Let $\mathcal{S}$ be a neutrosophic set in neutrosophic topological space $\mathcal{W}$. Then

- $\mathcal{N}int(\mathcal{S})$=$\cup\{\mathcal{K} : \mathcal{K}$ is an NOS in $\mathcal{W}$ and $\mathcal{K} \subseteq \mathcal{S}\}$ is called a neutrosophic interior of $\mathcal{S}$.

- $\mathcal{N}cl(\mathcal{S})$=$\cap\{\mathcal{L} : \mathcal{L}$ is an NCS in $\mathcal{W}$ and $\mathcal{L} \supseteq \mathcal{S}\}$ is called a neutrosophic closure of $\mathcal{S}$.

Definition 2.6. [2] A subset $\mathcal{W}$ of a neutrosophic space $(\mathcal{W}, \varsigma)$ is called

- a neutrosophic pre-open set(NP-OS) if $\mathcal{S} \subseteq \mathcal{N}int(\mathcal{N}cl(\mathcal{S}))$, and a neutrosophic pre-closed set(NP-CS) if $\mathcal{N}cl(\mathcal{N}int(\mathcal{S})) \subseteq \mathcal{S}$,
- a neutrosophic semi pre-open set(NSP-OS) if $\mathcal{S} \subseteq \mathcal{N}cl(\mathcal{N}int(\mathcal{N}cl(\mathcal{S})))$, and a neutrosophic semi pre-closed set(NSP-CS) if $\mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(\mathcal{S}))) \subseteq \mathcal{S}$,
- a neutrosophic regular-open set(NR-OS) if $\mathcal{S} = \mathcal{N}int(\mathcal{N}cl(\mathcal{S}))$, and a neutrosophic regular-closed set(NR-CS) if $\mathcal{S} = \mathcal{N}cl(\mathcal{N}int(\mathcal{S}))$,
- a neutrosophic semi-open set(NS-OS) if $\mathcal{S} \subseteq \mathcal{N}cl(\mathcal{N}int(\mathcal{S}))$, and a neutrosophic semi-closed set(NS-CS) if $\mathcal{N}int(\mathcal{N}cl(\mathcal{S})) \subseteq \mathcal{S}$,
- a neutrosophic α-open set($\mathcal{N}_\alpha$-OS) if $\mathcal{S} \subseteq \mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(\mathcal{S})))$, and a neutrosophic α-closed set($\mathcal{N}_\alpha$-CS) if $\mathcal{N}cl(\mathcal{N}int(\mathcal{N}cl(\mathcal{S}))) \subseteq \mathcal{S}$.

Definition 2.7. A subset $\mathcal{S}$ of a space $(\mathcal{S}, \varsigma)$ is called

- a neutrosophic generalized closed set[3](briefly $\mathcal{N}_g$-CS) if $\mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{K}$ whenever $\mathcal{S} \subseteq \mathcal{K}$ and $\mathcal{K}$ is NOS in $(\mathcal{W}, \varsigma)$. The complement of a $\mathcal{N}_g$- closed set is called a $\mathcal{N}_g$-open set($\mathcal{N}_g$-OS).
- a neutrosophic generalized semi-closed set[7](briefly $\mathcal{N}_{gs}$-CS) if $\mathcal{N}scl(\mathcal{S}) \subseteq \mathcal{K}$ whenever $\mathcal{S} \subseteq \mathcal{K}$ and $\mathcal{K}$ is NOS in $(\mathcal{W}, \varsigma)$.
- a neutrosophic α-generalized closed set[4](briefly $\mathcal{N}_{\alpha g}$-CS) if $\mathcal{N}\alpha cl(\mathcal{S}) \subseteq \mathcal{K}$ whenever $\mathcal{S} \subseteq \mathcal{K}$ and $\mathcal{K}$ is NOS in $(\mathcal{W}, \varsigma)$.
- a neutrosophic generalized pre closed set[1](briefly $\mathcal{N}_{gp}$-CS) set if $\mathcal{N}pcl(\mathcal{S}) \subseteq \mathcal{K}$ whenever $\mathcal{S} \subseteq \mathcal{K}$ and $\mathcal{K}$ is NOS in $(\mathcal{W}, \varsigma)$.
- a neutrosophic generalized semi pre-closed set(briefly $\mathcal{N}_{gsp}$-CS) if $\mathcal{N}spcl(\mathcal{S}) \subseteq \mathcal{K}$ whenever $\mathcal{S} \subseteq \mathcal{K}$ and $\mathcal{K}$ is NOS in $(\mathcal{W}, \varsigma)$.
- a neutrosophic generalized pre regular closed set[5](briefly $\mathcal{N}_{gpr}$-CS) if $\mathcal{N}pcl(\mathcal{S}) \subseteq \mathcal{K}$ whenever $\mathcal{S} \subseteq \mathcal{K}$ and $\mathcal{K}$ is NR-OS in $(\mathcal{W}, \varsigma)$.
- a neutrosophic generalized α-closed set[10](briefly $\mathcal{N}_{g\alpha}$-CS) if $\mathcal{N}\alpha cl(\mathcal{S}) \subseteq \mathcal{K}$ whenever $\mathcal{S} \subseteq \mathcal{K}$ and $\mathcal{K}$ is $\mathcal{N}_\alpha$-OS in $(\mathcal{W}, \varsigma)$.

Throughout this paper neutrosophic $^*g\alpha$-open set and neutrosophic $^*g\alpha$- closed set is denoted by $\mathcal{N}_{^*g\alpha}$-OS and $\mathcal{N}_{^*g\alpha}$-CS respectively.

3. Properties of $\mathcal{N}_{^*g\alpha}$-CS

Definition 3.1. A subset $\mathcal{W}$ of $(\mathcal{W}, \varsigma)$ is known as $\mathcal{N}_{^*g\alpha}$-CS if $\mathcal{U} \supseteq \mathcal{N}_{cl}(\mathcal{S})$ whenever $\mathcal{U} \supseteq \mathcal{S}, \mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS in $(\mathcal{W}, \varsigma)$. The complement of $\mathcal{N}_{^*g\alpha}$-CS is called a $\mathcal{N}_{^*g\alpha}$-OS.

Theorem 3.2. (1) *Any NCS is $\mathcal{N}_{^*g\alpha}$-CS.*
(2) *Any $\mathcal{N}_{^*g\alpha}$-CS is $\mathcal{N}_g$-CS.*
(3) *Any $\mathcal{N}_{^*g\alpha}$-CS is $\mathcal{N}_{g\alpha}$-CS.*
(4) *Any $\mathcal{N}_{^*g\alpha}$-CS is $\mathcal{N}_{gp}$-CS.*
(5) *Any $\mathcal{N}_{^*g\alpha}$-CS is $\mathcal{N}_{gs}$-CS.*
(6) *Any $\mathcal{N}_{^*g\alpha}$-CS is $\mathcal{N}_{\alpha g}$-CS.*
(7) *Any $\mathcal{N}_{^*g\alpha}$-CS is $\mathcal{N}_{gsp}$-CS.*
(8) *Any $\mathcal{N}_{^*g\alpha}$-CS is $\mathcal{N}_{gpr}$-CS.*

Proof. (1) *Let $\mathcal{U} \supseteq \mathcal{S}$, where $\mathcal{U}$ is a $\mathcal{N}_{g\alpha}$-OS in $(\mathcal{W}, \varsigma)$. Since $\mathcal{S}$ is NCS, then $\mathcal{U} \supseteq \mathcal{S} = \mathcal{N}cl(\mathcal{S})$. Therefore $\mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. Hence $\mathcal{S}$ is $\mathcal{N}_{^*g\alpha}$-CS.*
(2) *Let $\mathcal{U} \supseteq \mathcal{S}$, where $\mathcal{U}$ is NOS in $(\mathcal{W}, \varsigma)$. Since every NOS is $\mathcal{N}_{g\alpha}$-OS, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS in $(\mathcal{W}, \varsigma)$. Since $\mathcal{S}$ is $\mathcal{N}_{^*g\alpha}$-CS, $\mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. Hence $\mathcal{S}$ is $\mathcal{N}_g$-CS.*
(3) *Let $\mathcal{U} \subseteq \mathcal{S}$, where $\mathcal{U}$ is $\mathcal{N}_\alpha$-OS in $(\mathcal{W}, \varsigma)$. Since every $\mathcal{N}_\alpha$-OS is $\mathcal{N}_{g\alpha}$-OS, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$- OS in $(\mathcal{W}, \varsigma)$. Since $\mathcal{S}$ is $\mathcal{N}_{^*g\alpha}$-CS, $\mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. But $\mathcal{N}\alpha cl(\mathcal{S}) \subseteq \mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$. Therefore $\mathcal{S}$ is $\mathcal{N}_{g\alpha}$-CS.*
(4) *Let $\mathcal{U} \supseteq \mathcal{S}$, where $\mathcal{U}$ is NOS in $(\mathcal{W}, \varsigma)$. Since every NOS is $\mathcal{N}_{g\alpha}$-OS, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS in $(\mathcal{W}, \varsigma)$. Since $\mathcal{S}$ is $\mathcal{N}_{^*g\alpha}$-CS, $\mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. But $\mathcal{N}pcl(\mathcal{S}) \subseteq \mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$. Therefore $\mathcal{S}$ is $\mathcal{N}_{gp}$-CS.*
(5) *Let $\mathcal{U} \supseteq \mathcal{S}$, where $\mathcal{U}$ is NOS in $(\mathcal{W}, \varsigma)$. Since every NOS is $\mathcal{N}_{g\alpha}$-OS, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS in $(\mathcal{W}, \varsigma)$. Since $\mathcal{S}$ is $\mathcal{N}_{^*g\alpha}$-CS, $\mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. But $\mathcal{N}scl(\mathcal{S}) \subseteq \mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$. Therefore $\mathcal{S}$*

is $\mathcal{N}_{gs}$-CS.
(6) *Let $\mathcal{S} \subseteq \mathcal{U}$, where $\mathcal{U}$ is a neutrosophic OS in $(\mathcal{W}, \varsigma)$. Since every neutrosophic OS is $\mathcal{N}_{g\alpha}$-OS, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS. Since $\mathcal{S}$ is $\mathcal{N}_{*g\alpha}$-CS, $\mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$. But $\mathcal{N}\alpha cl(\mathcal{S}) \subseteq \mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$. Therefore $\mathcal{S}$ is $\mathcal{N}_{\alpha g}$-CS.*
(7) *Let $\mathcal{U} \supseteq \mathcal{S}$, where $\mathcal{U}$ is a NOS in $(\mathcal{W}, \varsigma)$. Since every NOS is $\mathcal{N}_{g\alpha}$-OS, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS. Since $\mathcal{S}$ is $\mathcal{N}_{*g\alpha}$-CS, $\mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. But $\mathcal{N}spcl(\mathcal{S}) \subseteq \mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$. Therefore $\mathcal{S}$ is $\mathcal{N}_{gsp}$-CS.*
(8) *Let $\mathcal{U} \supseteq \mathcal{S}$, where $\mathcal{U}$ is a NROS in $(\mathcal{W}, \varsigma)$. Since every NROS is $\mathcal{N}_{g\alpha}$-OS, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS. Since $\mathcal{S}$ is $\mathcal{N}_{*g\alpha}$-CS, $\mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. But $\mathcal{N}pcl(\mathcal{S}) \subseteq \mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$. Therefore $\mathcal{S}$ is $\mathcal{N}_{gpr}$-CS.*
The following example demonstrate reversal of this theorem is not necessary true

Example 3.3. (1) Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.3, 0.5, 0.3)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.3, 0.5, 0.6)\rangle$ is a $\mathcal{N}_{g\alpha}$-OS and
$\mathcal{A}_3$=$\langle a, (0.2, 0.6, 0.7)\rangle$ is a $\mathcal{N}_{*g\alpha}$-CS but it is not NCS of a NTS $(\mathcal{W}, \varsigma)$. Since $\mathcal{N}cl(\mathcal{S}_3) \neq \mathcal{S}_3$.
(2) Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.8, 0.5, 0.7)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.7, 0.4, 0.9)\rangle$ is a $\mathcal{N}_{g\alpha}$-OS and
$\mathcal{S}_3$=$\langle a, (0.7, 0.7, 0.8)\rangle$ is a $\mathcal{N}_g$-CS but it is not $\mathcal{N}_{*g\alpha}$-CS of a NTS $(\mathcal{W}, \varsigma)$. Since $\mathcal{N}cl(\mathcal{S}_3) \nsubseteq \mathcal{S}_2$, where $\mathcal{S}_3 \subseteq \mathcal{S}_2$.
(3) Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.4, 0.6, 0.7)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.7, 0.4, 0.6)\rangle$ is a $\mathcal{N}_{g\alpha}$-OS and
$\mathcal{S}_3$=$\langle a, (0.6, 0.6, 0.7)\rangle$ is a $\mathcal{N}_{g\alpha}$-CS but it is not $\mathcal{N}_{*g\alpha}$-CS of a NTS $(\mathcal{W}, \varsigma)$. Since $\mathcal{N}cl(\mathcal{S}_3) \nsubseteq \mathcal{S}_2$, where $\mathcal{S}_3 \subseteq \mathcal{S}_2$.
(4) Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.4, 0.4, 0.6)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.2, 0.2, 0.6)\rangle$ is a $\mathcal{N}_{gp}$-CS but it is not $\mathcal{N}_{*g\alpha}$-CS of a NTS $(\mathcal{W}, \varsigma)$.
(5) Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.8, 0.6, 0.7)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.7, 0.5, 0.7)\rangle$ is both $\mathcal{N}_{g\alpha}$ and $\mathcal{N}_{gs}$-CS but it is not $\mathcal{N}_{*g\alpha}$-CS of a neutrosophic topological space $(\mathcal{W}, \varsigma)$. Since $\mathcal{N}cl(\mathcal{S}_2) \nsubseteq (\mathcal{S}_2)^c$, where $\mathcal{S}_2 \subseteq (\mathcal{S}_2)^c$.
(6) Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{A}_1, 1_{\mathcal{N}}\}$ be a neutrosophic topology on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.2, 0.2, 0.3)\rangle$, which is the neutrosophic OS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.6, 0.2, 0.4)\rangle$ is both $\mathcal{N}_{g\alpha}$ and $\mathcal{N}_{\alpha g}$-CS but it is not $\mathcal{N}_{*g\alpha}$-CS of a NTS $(\mathcal{W}, \varsigma)$.
(7) Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.5, 0.5, 0.6)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.6, 0.7, 0.7)\rangle$ is a $\mathcal{N}_{gsp}$-CS but it is not $\mathcal{N}_{*g\alpha}$-CS of a NTS $(\mathcal{W}, \varsigma)$.
(8) Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.5, 0.3, 0.6)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.2, 0.1, 0.7)\rangle$ is a $\mathcal{N}_{gpr}$-CS but it is not $\mathcal{N}_{*g\alpha}$-CS of a NTS $(\mathcal{W}, \varsigma)$.

Remark 3.4. $\mathcal{N}_{*g\alpha}$-closedness is independent of neutrosophic semi-closedness and neutrosophic α-closedness.

Example 3.5. Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.3, 0.5, 0.3)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.3, 0.5, 0.6)\rangle$ is a $\mathcal{N}_{g\alpha}$-OS and
$\mathcal{S}_3$=$\langle a, (0.2, 0.6, 0.7)\rangle$ is a $\mathcal{N}_{*g\alpha}$-CS but it is not neutrosophic semi-CS and neutrosophic α-CS of a NTS $(\mathcal{W}, \varsigma)$.

Example 3.6. Let $\mathcal{W}$=$\{a\}$, ς=$\{0_{\mathcal{N}}, \mathcal{S}_1, 1_{\mathcal{N}}\}$ be a NT on $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_1$=$\langle a, (0.8, 0.5, 0.7)\rangle$, which is the NOS in $(\mathcal{W}, \varsigma)$.
$\mathcal{S}_2$=$\langle a, (0.7, 0.4, 0.9)\rangle$ is a $\mathcal{N}_{g\alpha}$-OS and
$\mathcal{S}_3$=$\langle a, (0.7, 0.7, 0.8)\rangle$ is both neutrosophic semi-CS and neutrosophic α-CS but it is not $\mathcal{N}_{*g\alpha}$-CS of a NTS $(\mathcal{W}, \varsigma)$. Since $\mathcal{N}cl(\mathcal{S}_3) \nsubseteq \mathcal{S}_2$, where $\mathcal{S}_3 \subseteq \mathcal{S}_2$.

Theorem 3.7. (1) *If $\mathcal{S}$ is a $\mathcal{N}_{*g\alpha}$-CS and $\mathcal{S} \subseteq \mathcal{R} \subseteq \mathcal{N}cl(\mathcal{S})$, then $\mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-CS.*
(2) *A subset $\mathcal{S}$ of $(\mathcal{W}, \varsigma)$ is $\mathcal{N}_{*g\alpha}$-OS iff $\mathcal{N}int(\mathcal{S}) \supseteq \mathcal{U}$, whenever $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-CS and $\mathcal{S} \supseteq \mathcal{U}$.*
(3) *The union of two $\mathcal{N}_{*g\alpha}$-CS's is a $\mathcal{N}_{*g\alpha}$-CS.*
(4) *The intersection of two $\mathcal{N}_{*g\alpha}$-CS's is a $\mathcal{N}_{*g\alpha}$-CS.*
(5) *The intersection of a NOS and a $\mathcal{N}_{*g\alpha}$-OS is a $\mathcal{N}_{*g\alpha}$-OS.*
(6) *The union of a NOS and a $\mathcal{N}_{*g\alpha}$-OS is a $\mathcal{N}_{*g\alpha}$-OS.*
Proof.

(1) *Let $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS of $(\mathcal{W}, \varsigma)$ such that $\mathcal{U} \supseteq \mathcal{R}$. Then $\mathcal{U} \supseteq \mathcal{S}$. Since $\mathcal{S}$ is a $\mathcal{N}_{*g\alpha}$-closed, $\mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. Now $\mathcal{N}cl(\mathcal{R}) \subseteq \mathcal{N}cl(\mathcal{N}cl(\mathcal{S})) = \mathcal{U} \supseteq \mathcal{N}cl(\mathcal{S})$. Therefore $\mathcal{R}$ is also $\mathcal{N}_{*g\alpha}$-CS of $(\mathcal{W}, \varsigma)$.*
(2) *Let $\mathcal{S}$ be a $\mathcal{N}_{*g\alpha}$-OS and $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-CS such that $\mathcal{S} \supseteq \mathcal{U}$ implies $(\mathcal{U})^c \supseteq (\mathcal{S})^c$, $(\mathcal{S})^c$ is $\mathcal{N}_{*g\alpha}$-CS. Thus $\mathcal{N}cl((\mathcal{S})^c) \subseteq (\mathcal{U})^c$ implies $\mathcal{U} \subseteq (\mathcal{N}cl((\mathcal{S})^c))^c$. But $(\mathcal{N}cl((\mathcal{S})^c))^c = \mathcal{N}int(\mathcal{S})$. So $\mathcal{U} \subseteq \mathcal{N}int(\mathcal{S})$. Conversely, if $\mathcal{S}$ is subset such that $\mathcal{U} \subseteq \mathcal{N}int(\mathcal{S})$. Whenever $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-CS and $\mathcal{U} \subseteq \mathcal{S}$. We show that $(\mathcal{S})^c$ is $\mathcal{N}_{*g\alpha}$- CS. Let $(\mathcal{S})^c \subseteq (\mathcal{U})^c$, Where $(\mathcal{U})^c$ is $\mathcal{N}_{g\alpha}$ - OS. By assumption we must have $(\mathcal{N}int(\mathcal{S}))^c \subseteq \mathcal{U}$. Now $(\mathcal{N}int(\mathcal{S}))^c = \mathcal{N}cl(\mathcal{S})^c$ which implies $\mathcal{N}cl(\mathcal{S})^c \subseteq \mathcal{U}$ and $(\mathcal{S})^c$ is $\mathcal{N}_{*g\alpha}$-CS.*
(3) *Let $\mathcal{S}$ and $\mathcal{R}$ are $\mathcal{N}_{*g\alpha}$-CS's. Let $\mathcal{S} \cup \mathcal{R} \subseteq \mathcal{U}$, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$ -OS. Since $\mathcal{S}$ and $\mathcal{R}$ are $\mathcal{N}_{*g\alpha}$-CS's, $\mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$ and $\mathcal{N}cl(\mathcal{R}) \subseteq \mathcal{U}$. This implies $\mathcal{N}cl(\mathcal{S} \cup \mathcal{R}) = \mathcal{N}cl(\mathcal{S}) \cup \mathcal{N}cl(\mathcal{R}) \subseteq \mathcal{U} \Rightarrow \mathcal{N}cl(\mathcal{S} \cup \mathcal{R}) \subseteq \mathcal{U}$. Therefore $\mathcal{S} \cup \mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-CS's.*
(4) *Let $\mathcal{S}$ and $\mathcal{R}$ are $\mathcal{N}_{*g\alpha}$-CS's. Let $\mathcal{S} \cap \mathcal{R} \subseteq \mathcal{U}$, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$-OS. Since $\mathcal{S}$ and $\mathcal{R}$ are $\mathcal{N}_{*g\alpha}$-CS's, $\mathcal{N}cl(\mathcal{S}) \subseteq \mathcal{U}$ and $\mathcal{N}cl(\mathcal{R}) \subseteq \mathcal{U}$. This implies $\mathcal{N}cl(\mathcal{S} \cap \mathcal{R}) \subseteq \mathcal{N}cl(\mathcal{S}) \cap \mathcal{N}cl(\mathcal{R}) \subseteq \mathcal{U} \Rightarrow \mathcal{N}cl(\mathcal{S} \cap \mathcal{R}) \subseteq \mathcal{U}$. Therefore $\mathcal{S} \cap \mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-CS's.*
(5) *Let $\mathcal{S}$ be a NOS and $\mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-OS. Let $\mathcal{U} \subseteq (\mathcal{S} \cap \mathcal{R})$, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$–CS. Since every NOS is $\mathcal{N}_{*g\alpha}$-OS, $\mathcal{S}$ and $\mathcal{R}$ are $\mathcal{N}_{*g\alpha}$-OS, $\mathcal{U} \subseteq \mathcal{N}int(\mathcal{S})$ and $\mathcal{U} \subseteq \mathcal{N}int(\mathcal{R})$. This implies $\mathcal{U} \subseteq \mathcal{N}int(\mathcal{S}) \cap \mathcal{N}int(\mathcal{R}) = \mathcal{N}int(\mathcal{S} \cap \mathcal{R}) \Rightarrow \mathcal{U} \subseteq \mathcal{N}int(\mathcal{S} \cap \mathcal{R})$. Therefore $\mathcal{S} \cap \mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-OS.*
(6) *Let $\mathcal{S}$ be a NOS and $\mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-OS. Let $\mathcal{U} \subseteq (\mathcal{S} \cup \mathcal{R})$, $\mathcal{U}$ is $\mathcal{N}_{g\alpha}$–CS. Since every NOS is $\mathcal{N}_{*g\alpha}$-OS, $\mathcal{S}$ and $\mathcal{R}$ are $\mathcal{N}_{*g\alpha}$-OS's, $\mathcal{U} \subseteq \mathcal{N}int(\mathcal{S})$ and $\mathcal{U} \subseteq \mathcal{N}int(\mathcal{R})$. This implies $\mathcal{U} \subseteq \mathcal{N}int(\mathcal{S}) \cup \mathcal{N}int(\mathcal{R}) \subseteq \mathcal{N}int(\mathcal{S} \cup \mathcal{R}) \Rightarrow \mathcal{U} \subseteq \mathcal{N}int(\mathcal{S} \cup \mathcal{R})$. Therefore $\mathcal{S} \cup \mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-OS.*

4. Conclusion

This article defined $\mathcal{N}_{^*g\alpha}$-Closed Set in neutrosophic topological spaces and discussed some of their properties. Also further in future, using this set we can derive $\mathcal{N}_{^*g\alpha}$-interior, $\mathcal{N}_{^*g\alpha}$-closure, $\mathcal{N}_{^*g\alpha}$-border and $\mathcal{N}_{^*g\alpha}$-frontier in neutrosophic topological spaces. In addition, this set can be used to derive few more new functions of $\mathcal{N}_{^*g\alpha}$-continuous and $\mathcal{N}_{^*g\alpha}$-homeomorphisms in neutrosophic topological spaces are established.

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