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MORE CHARACTERIZATIONS ABOUT SOFT SETS AND SOFT CONTINUITY

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Abstract. We introduce and study the indicator soft sets $(Ind(A), \mathcal{P})$ where $A \subset \mathcal{P}$ and $\mathcal{P}$ is the set of parameters. We use the indicator soft sets to define the indicator soft topology $Ind(\tau)$ where τ is any topology on the set of parameters $\mathcal{P}$. We show that τ and $Ind(\tau)$ have the same corresponding properties: compactness, Lindelöf property, second-countable, separation axioms. We proved that a soft mapping $fpu : (\mathcal{U}, Ind(\tau), \mathcal{P}) \to (\mathcal{U}', Ind(\tau'), \mathcal{P}')$ is soft continuous (quasi-continuous, open, closed, soft homeomorphism) iff $p : (\mathcal{P}, \tau) \to (\mathcal{P}', \tau')$ is continuous (quasi-continuous, open, closed, homeomorphism). We use these results to generate counter examples for soft topological spaces. For example, we gave an example to show that soft quasi-continuous mapping are not soft continuous, another example to show that soft $T_3 - space$ are not soft T_4, and an example to show that none of the soft separation axioms are preserved under soft open mappings.

1. Preliminaries

The soft set theory is introduced In 1999 (see [12]) with various applications of the new theory. In 2003 the notation of soft subset, complement, union and intersections (see [11])are defined and studied . In 2010 soft topology is introduced by [17] and [3]. Other papers about soft topology appeared in 2011 (see [7] and [18]).

Soft points first defined in 2011 as points in the universe with certain criteria, see [17], and still in use by many authors, in 2012 [24] introduced soft functions and a new definition for soft points. Soft compactness first introduced by [24] and soft separation axioms by [8].

Up to the present, soft sets and topologies are still a very active area for research in the theoretical aspect specially concerning soft separation axioms [9],[14] and [27], and

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in applications, specially on vagueness theory and decision making, see for example [15] and [5].

Definition 1.1. [4] Consider the parameters set $\mathcal{P}$ and the universe $\mathcal{U}$ and $2^{\mathcal{U}}$ (all subsets of $\mathcal{U}$).

(1) The ordered pair $(F,\mathcal{P})$ is said a soft set if
$$(F,\mathcal{P}) = \{(e,F(e)); e \in \mathcal{P}\}$$
where $F:\mathcal{P}\to 2^{\mathcal{U}}$.
$SS(\mathcal{U})_{\mathcal{P}}$ stands for all soft set on $\mathcal{U}$.

(2) A soft set $(F,\mathcal{P})$ is a *soft subset* of $(G,\mathcal{P})$ (in symbols $(F,\mathcal{P})\widetilde{\subset}(G,\mathcal{P})$) if $F(x)\subset G(x)$ for all $x\in\mathcal{P}$.

(3) The *soft complement* of $(F,\mathcal{P})$ (in symbols $(F,\mathcal{P})^{\widetilde{c}}$) is the soft set$(F^{\widetilde{c}},\mathcal{P})$ such that $F^{\widetilde{c}}(x)=\mathcal{U}\setminus f(x)$.

(4) Let $(F,\mathcal{P})$, $(G,\mathcal{P})$ be two soft sets in $SS(\mathcal{U})_{\mathcal{P}}$. The *soft union* of $(F,\mathcal{P})$ and $(G,\mathcal{P})$ (denoted by $(F,\mathcal{P})\widetilde{\cup}(G,\mathcal{P})$) is the soft set $(F\widetilde{\cup}G,\mathcal{P})$ such that $(F\widetilde{\cup}G)(x)=F(x)\cup G(x)$.

(5) Let $(F,\mathcal{P})$, $(G,\mathcal{P})$ be two soft sets in $SS(\mathcal{U})_{\mathcal{P}}$. The *soft intersection* of $(F,\mathcal{P})$ and $(G,\mathcal{P})$ (denoted by $(F,\mathcal{P})\widetilde{\cap}(G,\mathcal{P})$) is the soft set $(F\widetilde{\cap}G,\mathcal{P})$ such that $(F\widetilde{\cap}G)(x)=F(x)\cap G(x)$.

(6) Let $(F,\mathcal{P})$, $(G,\mathcal{P})$ be two soft sets in $SS(\mathcal{U})_{\mathcal{P}}$. The *soft difference* of $(F,\mathcal{P})$ and $(G,\mathcal{P})$ (denoted by $F\widetilde{\setminus}G$) is the soft set $(F\widetilde{\setminus}G,\mathcal{P})$ such that $(F\widetilde{\setminus}G)(x)=F(x)\setminus G(x)$.

(7) $(F,\mathcal{P})$ and $(G,\mathcal{P})$ are said *disjoint* iff $F(x)\cap G(x)=\emptyset$ for every $x\in\mathcal{P}$.

(8) $\mathcal{U}_{\mathcal{P}}$ stands for the *universal soft set* $(F,\mathcal{P})$ where $F(e)=\mathcal{U}$ for every $e\in\mathcal{P}$.

(9) $\emptyset_{\mathcal{P}}$ stands for the *Empty soft* set $(F,\mathcal{P})$ where $F(e)=\emptyset$ for every $e\in\mathcal{P}$.

Definition 1.2. [24] $(F,\mathcal{P})\in SS(\mathcal{U})_{\mathcal{P}}$ is said a *soft point* in $\mathcal{U}_{\mathcal{P}}$ (in symbols e_F where $e\in\mathcal{P}$) if: $F(x)\neq\emptyset$ iff $x=e$.

Definition 1.3. [24] The soft point e_F is said to be in the soft set $(G,\mathcal{P})$ (denoted by $e_F\widetilde{\in}(G,B))$ iff $F(e)\subset G(e)$.

Proposition 1.4. [24] If $e_F\widetilde{\in}(G,B)$, then $e_F\widetilde{\notin}(G,B)^{\widetilde{c}}$

Definition 1.5. [24] Two soft points e_F and e'_G are said *distinct* (we write $e_F\neq e_G$) if and only $F(x)\cap G(x)=\emptyset$ for every $x\in\mathcal{P}$.

Definition 1.6. [24] Consider $SS(\mathcal{U})_{\mathcal{P}}$ and $SS(V)_{\mathcal{P}'}$. Let $u:\mathcal{U}\to V$ and $p:\mathcal{P}\to\mathcal{P}'$ be mappings. Then $f_{pu}:SS(\mathcal{U})_{\mathcal{P}}\to SS(V)_{\mathcal{P}'}$ is called a *soft mapping* if it is defined as follows:

(1) For any soft set $(F,\mathcal{P})\in SS(\mathcal{U})_{\mathcal{P}}$ the image of $(F,\mathcal{P})$ under f_{pu} (denoted by $f_{pu}(F,\mathcal{P})$) is the soft set $(f_{pu}(F),\mathcal{P}')\in SS(V)_{\mathcal{P}'}$ such that for every $b\in\mathcal{P}'$ we have $f_{pu}(F)(b)=\underset{x\in p^{-1}(b)}{\cup}u(F(x))$ if $b\in p(\mathcal{P})$, and $f_{pu}(F)(b)=\emptyset$ if $b\notin p(\mathcal{P})$.

(2) For any soft set $(G,\mathcal{P}')\in SS(V)_{\mathcal{P}'}$, the inverse image of $(G,\mathcal{P})$ under f_{pu} (denoted by $f_{pu}^{-1}(G,\mathcal{P})$) is the soft set $(f_{pu}^{-1}(G),\mathcal{P})\in SS(\mathcal{U})_{\mathcal{P}}$ such that for every $a\in\mathcal{P}$ we have $f_{pu}^{-1}(G)(a)=u^{-1}(G(p(a)))$.

The properties of soft functions can be found in [24].

Definition 1.7. [3] A subcollection $\mathcal{T}$ from $SS(\mathcal{U})_{\mathcal{P}}$ is called a soft topology if it satisfies the following:

(1) $\emptyset_{\mathcal{P}}, \mathcal{U}_{\mathcal{P}} \in \mathcal{T}$.
(2) The soft intersection of two elements from $\mathcal{T}$ is an element of $\mathcal{T}$.
(3) The soft union of elements from $\mathcal{T}$ is an element of $\mathcal{T}$.

The triple $(\mathcal{U}, \mathcal{T}, \mathcal{P})$ is said a *soft topological space*. $(F, \mathcal{P})$ is called *soft open* if $(F, \mathcal{P}) \in \mathcal{T}$, and it is called *soft closed* if $(F, \mathcal{P})^{\widetilde{c}} \in \mathcal{T}$.

Definition 1.8. consider $(\mathcal{U}, \mathcal{T}, \mathcal{P})$ and let $(F, \mathcal{P})$ be any soft set in $SS(\mathcal{U})_{\mathcal{P}}$. Then:

(1) [17] The soft closure of $(F, \mathcal{P})$ in $\mathcal{T}$ is defined by $Cl_{\mathcal{T}}(F, \mathcal{P}) = \widetilde{\bigcap}\{(G, \mathcal{P}); (G, \mathcal{P})^{\widetilde{c}} \text{in} \mathcal{T} \text{and} (G, \mathcal{P}) \widetilde{\subset} (F, \mathcal{P})\}$.
(2) [17] The soft interior of $(F, \mathcal{P})$ in $\mathcal{T}$ is defined by $Int_{\mathcal{T}}(F, \mathcal{P}) = \widetilde{\bigcup}\{(G, \mathcal{P}); (G, \mathcal{P})$ is soft open and soft contained in $(F, \mathcal{P})\}$.
(3) [7]The soft boundary of $(F, \mathcal{P})$ in $\mathcal{T}$ is defined by $\partial_{\mathcal{T}}(F, \mathcal{P}) = Cl_{\mathcal{T}}(F, \mathcal{P}) \widetilde{\cap} Cl_{\mathcal{T}}((F, \mathcal{P})^{\widetilde{c}})$.

Definition 1.9. [8] $(\mathcal{U}, \mathcal{T}, \mathcal{P})$ is said to be:

(1) A soft $T_0 - space\ iff$ for any two distinct soft points e_F and e'_G in $\mathcal{U}_{\mathcal{P}}$, there is $(F, \mathcal{P})$ in $\mathcal{T}$ such that $e_F \widetilde{\in} (F, \mathcal{P})$ and $e'_G \widetilde{\notin} (F, \mathcal{P})$ or $e_F \widetilde{\notin} (F, \mathcal{P})$ and $e'_G \widetilde{\in} (F, \mathcal{P})$.
(2) A soft $T_1 - space\ iff$ for any e_F and e'_G in $\mathcal{U}_{\mathcal{P}}$ distinct, there are $(F, \mathcal{P})$ and $(G, \mathcal{P})$ in $\mathcal{T}$ with $e_F \widetilde{\in} (F, \mathcal{P})$ and $e'_G \widetilde{\notin} (F, \mathcal{P})$.
(3) A soft $T_2 - space\ iff$ for any e_F and e'_G in $\mathcal{U}_{\mathcal{P}}$ distinct, there are two disjoint soft open sets $(F, \mathcal{P})$ and $(G, \mathcal{P})$ with $e_F \widetilde{\in} (F, \mathcal{P})$ and $e'_G \widetilde{\in} (G, \mathcal{P})$.
(4) A soft $T_3 - space\ iff$ it is soft T_1 and *soft regular*: for any e_F (soft point) and any $(C, \mathcal{P})$ (soft closed) with $e_F \widetilde{\notin} (C, \mathcal{P})$ there are $(F, \mathcal{P})$ and $(G, \mathcal{P})$ (two soft disjoint open sets) such that $e_F \widetilde{\in} (F, \mathcal{P})$ and $(C, \mathcal{P}) \widetilde{\subset} (G, \mathcal{P})$.
(5) A soft $T_4 - space\ iff$ it is soft T_1 and *soft normal*: for any two disjoint soft closed sets $(C, \mathcal{P})$ and $(C', \mathcal{P})$ there are two soft disjoint open sets $(F, \mathcal{P})$ and $(G, \mathcal{P})$ such that $(C, \mathcal{P}) \widetilde{\subset} (F, \mathcal{P})$ and $(C', \mathcal{P}) \widetilde{\subset} (G, \mathcal{P})$.

Definition 1.10. [24] Let $(\mathcal{U}, \mathcal{T}, \mathcal{P})$ and $(\mathcal{U}', \mathcal{T}', \mathcal{P}')$ be tow soft topological spaces. and $u : \mathcal{U} \to \mathcal{U}'$ and $p : \mathcal{P} \to \mathcal{P}'$. The soft mapping $f_{pu} : SS(\mathcal{U})_{\mathcal{P}} \to SS(V)_{\mathcal{P}'}$ is said to be:

(1) $pu - continuous$ at $e_F \widetilde{\in} \mathcal{U}_{\mathcal{P}}$ if for each soft open set $(G, \mathcal{P}') \widetilde{\in} \mathcal{T}'$ soft containing $f_{pu}(e_F)$ there exists a soft open set $(F_1, \mathcal{P}) \widetilde{\in} \mathcal{T}$ soft containing e_F such that $f_{pu}(F_1, \mathcal{P}) \widetilde{\subset} (G, \mathcal{P}')$.
(2) $pu - continuous$ on $\mathcal{U}_{\mathcal{P}}$ if it is $pu - continuous$ at every $e_F \widetilde{\in} \mathcal{U}_{\mathcal{P}}$.

Theorem 1.11. [24] *Let $(\mathcal{U}, \mathcal{T}, \mathcal{P})$ and $(\mathcal{U}', \mathcal{T}', \mathcal{P}')$ be tow soft topological spaces. and $u : \mathcal{U} \to \mathcal{U}'$ and $p : \mathcal{P} \to \mathcal{P}'$. Then the following are equivalent:*

(1) The mapping $f_{pu} : SS(\mathcal{U})_{\mathcal{P}} \to SS(V)_{\mathcal{P}'}$ is $pu - continuous$.
(2) For every $(G, \mathcal{P}') \in \mathcal{T}'$, $f_{pu}^{-1}(G, \mathcal{P}') \in \mathcal{T}$.
(3) For every soft closed set $(G, \mathcal{P}')$ in $\mathcal{T}'$, $f_{pu}^{-1}(G, \mathcal{P}')$ is soft closed in $\mathcal{T}$.

Soft continuity can be defined by the means of sot points to be come the so called sp-coninuity, for more about sp-continuity one can consult [2]

Definition 1.12. [25] Let $(\mathcal{U}, \mathcal{T}, \mathcal{P})$ and $(\mathcal{U}', \mathcal{T}', \mathcal{P}')$ be tow soft topological spaces. and $u : \mathcal{U} \to \mathcal{U}'$ and $p : \mathcal{P} \to \mathcal{P}'$ be mappings. Then the soft mapping $f_{pu} : SS(\mathcal{U})_{\mathcal{P}} \to SS(V)_{\mathcal{P}'}$ is said to be:

(1) one-to-one (onto) iff p and u are one to one (onto).
(2) $pu-open$ iff for every $(F,\mathcal{P})\in\mathcal{T}$ we have $f_{pu}(F,\mathcal{P})\in\mathcal{T}'$.
(3) soft homeomorphism iff it is one-to-one , onto, soft continuous and soft open.
(4) $pu-closed$ iff for every soft closed (in $\mathcal{T}$) set $(F,\mathcal{P})$, $f_{pu}(F,\mathcal{P})$ is soft closed in $\mathcal{T}'$.

Definition 1.13. [24] Consider the space $(\mathcal{U},\mathcal{T},\mathcal{P})$ and the soft set $(F,\mathcal{P})$.

(1) A collection $\widetilde{\Psi}$ of soft open sets is said a *soft open cover* of $(A,\mathcal{P})$ iff $(A,\mathcal{P})\widetilde{\subset}\widetilde{\bigcup}\widetilde{\Psi}$; if $\widetilde{\Psi}$ is also finite, then it is said to be a *finite soft cover.* A *soft subcover* of $\widetilde{\Psi}$ is any subfamily $\widetilde{\Psi}^*$ from $\widetilde{\Psi}$ which is also a soft cover of $(A,\mathcal{P})$.
(2) $(F,\mathcal{P})$ is called *soft compact* iff every soft open cover of $(F,\mathcal{P})$ has a finite soft subcover.

2. Indicator Soft Sets

Consider the universe $\mathcal{U}$ and the parameters set $\mathcal{P}$. For every $A\subset\mathcal{P}$ We define the soft set $(Ind(A),\mathcal{P})$ by the function $Ind(A):\mathcal{P}\to 2^{\mathcal{U}}$ as follows:

$$Ind(A)(e)=\begin{cases}\mathcal{U} & \text{if } e\in A\\ \emptyset & \text{if } e\notin A\end{cases}$$

This soft set will be called *the indicator* of $A\subset\mathcal{P}$ in $SS(\mathcal{U})$. It is obvious that the indicator set of A is similar to the indicator function of A in fuzzy set where 1 is replaced by $\mathcal{U}$ and 0 by $\emptyset$.

Theorem 2.1. *Consider the universe $\mathcal{U}$ and the set of parameters $\mathcal{P}$. For every $A,B\subset\mathcal{P}$ We have:*

(1) $\widetilde{\bigcup}_{\alpha\in\Delta}(Ind(A_\alpha),\mathcal{P})=(Ind(\bigcup_{\alpha\in\Delta}A_\alpha),\mathcal{P})$.
(2) $\widetilde{\bigcap}_{\alpha\in\Delta}(Ind(A_\alpha),\mathcal{P})=(Ind(\bigcap_{\alpha\in\Delta}A_\alpha),\mathcal{P})$.
(3) $(Ind(A),\mathcal{P})\widetilde{\cap}(Ind(B),\mathcal{P})=\emptyset_{\mathcal{P}}$ iff $A\cap B=\emptyset$.
(4) $(Ind(A),\mathcal{P})\widetilde{\setminus}(Ind(B),\mathcal{P})=(Ind(A\setminus B),\mathcal{P})$.
(5) $(Ind(A),\mathcal{P})^{\widetilde{c}}=(Ind(A^c),\mathcal{P})$.
(6) $(Ind(\emptyset),\mathcal{P})=\emptyset_{\mathcal{P}}$.
(7) $(Ind(\mathcal{P}),\mathcal{P})=\mathcal{U}_{\mathcal{P}}$.
(8) For any soft point e_F where $e\in\mathcal{P}$, We have $e_F\widetilde{\in}(Ind(\{e\}),\mathcal{P})$.

Proof. We prove (4) . For every $e\in\mathcal{P}$ we have:

$$(Ind(A),\mathcal{P})^{\widetilde{c}}(e)=\begin{cases}\mathcal{U}\setminus\mathcal{U} & \text{if } e\in A\\ \mathcal{U}\setminus\emptyset & \text{if } e\notin A\end{cases}=\begin{cases}\emptyset & \text{if } e\in A\\ \mathcal{U} & \text{if } e\notin A\end{cases}=(Ind(A^c),\mathcal{P})(e)\quad\square$$

Theorem 2.2. *Let $\mathcal{U}$ be the initial universe, $\mathcal{P}$ be a fixed set of parameters. If τ is a topology on $\mathcal{P}$, then $Ind(\tau)=\{(Ind(A),\mathcal{P});A\in\tau\}$ is a soft topology on $SS(\mathcal{U})_{\mathcal{P}}$.*

Proof. (1) Since $\emptyset,\mathcal{P}\in\tau$, $(Ind(\emptyset),\mathcal{P})=\emptyset_{\mathcal{P}}\in Ind(\tau)$ and $(Ind(\mathcal{P}),\mathcal{P})=\mathcal{U}_{\mathcal{P}}\in Ind(\tau)$.

(2) Let $(Ind(A),\mathcal{P})$ and $(Ind(B),\mathcal{P})$ in $Ind(\tau)$. Then $(Ind(A),\mathcal{P})\widetilde{\cap}(Ind(B),\mathcal{P}) = (Ind(A\cap B),\mathcal{P}) \in Ind(\tau)$, since $A\cap B \in \tau$.

(3) Let $(Ind(A_\alpha),\mathcal{P}) \in Ind(\tau)$ for every $\alpha \in \Delta$. Then $\widetilde{\bigcup}_{\alpha\in\Delta}(Ind(A_\alpha),\mathcal{P}) = (Ind(\bigcup_{\alpha\in\Delta} A_\alpha),\mathcal{P}) \in \tau$, since $\bigcup_{\alpha\in\Delta} A_\alpha \in \tau$.

□

The topology $Ind(\tau)$ will be called the *indicator topology* generated on $SS(\mathcal{U})_\mathcal{P}$ by τ. Now, we will study the relations between τ and $(Ind(\tau))$.

Theorem 2.3. *Consider the universe $\mathcal{U}$, and the set of parameters $\mathcal{P}$. Let τ be a topology on $\mathcal{P}$, and $Ind(\tau)$ be the indicator topology generated on $SS(\mathcal{U})$ by τ. Then*

(1) $A \subset \mathcal{P}$ is open in τ iff $(Ind(A),\mathcal{P})$ is soft open in $Ind(\tau)$.
(2) $A \subset \mathcal{P}$ is closed in τ iff $(Ind(A),\mathcal{P})$ is soft closed in $Ind(\tau)$.
(3) $(F,\mathcal{P})$ is soft closed in $Ind(\tau)$ iff there is a closed (in τ) set G with $(F,\mathcal{P}) = (Ind(G),\mathcal{P})$.
(4) For every $A\subset\mathcal{P}$, $Cl_{Ind(\tau)}(Ind(A),\mathcal{P}) = (Ind(Cl_\tau(A),\mathcal{P}))$.
(5) For every $A\subset\mathcal{P}$, $Int_{Ind(\tau)}(Ind(A),\mathcal{P}) = (Ind(Int_\tau(A),\mathcal{P}))$.
(6) For every $A\subset\mathcal{P}$, $\partial_{Ind(\tau)}(Ind(A),\mathcal{P}) = (Ind(\partial_\tau(A),\mathcal{P}))$.

Proof. (1) Obvious!

(2) Obvious!

(3) $(Ind(G),\mathcal{P})$ is soft closed set in $Ind(\tau)$ for every G closed in τ. Now, suppose that $(F,\mathcal{P})$ is any soft closed set in $Ind(\tau)$. Then $(F,\mathcal{P}) = (Ind(A),\mathcal{P})^{\tilde{c}}$ for some $A\in\tau$. But $(Ind(A),\mathcal{P})^{\tilde{c}} = (Ind(A^c),\mathcal{P})$.

(4) From (3) we have soft closed sets in $Ind(\tau)$ are those of the form $(Ind(F),\mathcal{P})$ where F is closed in τ. So:

$$\begin{aligned}Cl_{Ind(\tau)}(Ind(A),\mathcal{P}) &= \widetilde{\bigcap}\{(Ind(F),\mathcal{P}); F \text{ is closed in } \tau, A\subset F\}.\\ &= (Ind(\bigcap\{F; F \text{ is closed in } \tau, A\subset F\}),\mathcal{P}).\\ &= (Ind(Cl_\tau(A)),\mathcal{P}).\end{aligned}$$

(5)

$$\begin{aligned}Int_{Ind(\tau)}(Ind(A),\mathcal{P}) &= \widetilde{\bigcup}\{(Ind(O),\mathcal{P}); O \text{ is open in } \tau, O\subset A\}.\\ &= (Ind(\bigcup\{O; O \text{ is open in } \tau, O\subset A\}),\mathcal{P}).\\ &= (Ind(Int_\tau(A)),\mathcal{P}).\end{aligned}$$

(6)

$$\begin{aligned}\partial_{Ind(\tau)}(Ind(A),\mathcal{P}) &= Cl_{Ind(\tau)}(Ind(A),\mathcal{P})\widetilde{\cap}Cl_{Ind(\tau)}[(Ind(A),\mathcal{P})^{\tilde{c}}].\\ &= (Ind(Cl_\tau(A)),\mathcal{P})\widetilde{\cap}Cl_{Ind(\tau)}(Ind(A^c),\mathcal{P}).\\ &= (Ind(Cl_\tau(A)),\mathcal{P})\widetilde{\cap}(Ind(Cl_\tau(A^c)),\mathcal{P}).\\ &= (Ind(Cl_\tau(A)\cap Cl_\tau A^c),\mathcal{P}).\\ &= (Ind(\partial_\tau(A)),\mathcal{P}).\end{aligned}$$

□

Theorem 2.4. *Consider the universe $\mathcal{U}$ and the parameters set $\mathcal{P}$. Let τ be a topology on $\mathcal{P}$, and $Ind(\tau)$ be the indicator topology generated on $SS(\mathcal{U})$ by τ. Then the only soft open covers of $\mathcal{U}_{\mathcal{P}}$ from $Ind(\tau)$ are those of the form $\widetilde{\Psi} = \{(Ind(A), \mathcal{P}); A \in \Psi\}$ where $\Psi \subset \tau$ is an open cover of $\mathcal{P}$.*

Proof. Let $\widetilde{\Psi}$ be and open cover of $\mathcal{U}_{\mathcal{P}}$ from $Ind(\tau)$. Then
$\widetilde{\Psi} = \{(Ind(A_\alpha), \mathcal{P}); A_\alpha$ is open in$\tau, \alpha \in \Delta\}$, and $\mathcal{U}_{\mathcal{P}} \widetilde{\subset} \widetilde{\cup} \widetilde{\Psi}$. Let $\Psi = \{A_\alpha; \alpha \in \Delta\}$ So for every $e \in \mathcal{P}$ we have
$\widetilde{\bigcup} \widetilde{\Psi}(e) = \widetilde{\bigcup}_{\alpha \in \Delta} (Ind(A_\alpha), \mathcal{P})(e) = \mathcal{U}_{\mathcal{P}}(e) = \mathcal{U}$. Since $(Ind(A_\alpha), \mathcal{P})(e)$ equals $\mathcal{U}$ or $\emptyset$, there is $\alpha_\circ \in \Delta$ such that $(Ind(A_{\alpha_\circ}), \mathcal{P})(e) = \mathcal{U}$. That means $e \in A_{\alpha_\circ}$, which implies that Ψ is an open cover of $\mathcal{P}$. □

Theorem 2.5. *Consider the universe $\mathcal{U}$ and the parameters set $\mathcal{P}$. Let τ be a topology on $\mathcal{P}$, and let $Ind(\tau)$ be the indicator topology generated on $SS(\mathcal{U})$ by τ. Then $(\mathcal{P}, \tau)$ is compact iff $(\mathcal{U}, Ind(\tau), \mathcal{P})$ is soft compact.*

Proof. Suppose $(\mathcal{P}, \tau)$ is compact and let $\widetilde{\Psi} = \{(Ind(A), \mathcal{P}); A \in \Psi\}$ be an open cover of $\mathcal{U}_{\mathcal{P}}$ from $Ind(\tau)$. Then Ψ is an open cover of $\mathcal{P}$. Since $(\mathcal{P}, \tau)$ is compact, there exist a finite subcover Ψ^* of $\mathcal{P}$ from ψ. It is clear that $\widetilde{\Psi^*} = \{(Ind(A), \mathcal{P}); A \in \Psi^*\}$ is a soft open subcover of $\mathcal{U}_{\mathcal{P}}$ from $\widetilde{\Psi}$. Conversely; can be done in the same manner. □

A soft topological space $(\mathcal{U}, \mathcal{T}, \mathcal{P})$ is said to be *soft second-countable* if it has a countable soft base. And it said *soft Lindelöf* if every soft open cover of $\mathcal{U}_{\mathcal{P}}$ has a countable soft subcover[23].

The following is obvious.

Theorem 2.6. *Consider the universe $\mathcal{U}$ and the parameters set $\mathcal{P}$. Let τ be a topology on $\mathcal{P}$, and let $Ind(\tau)$ be the indicator topology generated on $SS(\mathcal{U})$ by τ. Then*

(1) Bases for $Ind(\tau)$ are those of the form $\widetilde{\mathcal{B}} = \{B \in \mathcal{B}; \mathcal{B}$ is a base for $\tau\}$.
(2) Subbases for $Ind(\tau)$ are those of the form $\widetilde{\mathcal{S}} = \{B \in \mathcal{S}; \mathcal{S}$ is a subbase for $\tau\}$.
(3) $Ind(\tau)$ is soft Second-countable iff τ is soft second-countable.
(4) $Ind(\tau)$ is soft Lindelöf iff τ is Lindelöf.

Proof. Straightforward. □

The discrete topology $\mathcal{D}$ on $\mathbb{R}$ is second-countable (and so Lindelöf) but not compact, and any compact $T_1 - space$ with more that $2^{\aleph_\circ}$ points is not second countable. We can use these facts in soft set theory as follows.

Example 2.7.
(1) There is a soft second-countable space which is not soft compact. Consider the parameters set $\mathbb{R}$ and the universe $\mathcal{U}$ any nonempty set. Since $(\mathbb{R}, \mathcal{D})$. where$\mathcal{D}$ is the discrete topology on $\mathbb{R}$,is second countable(hence Lindelöf), $(\mathcal{U}, Ind(\mathcal{D}), \mathbb{R})$ is soft second countable and (hence soft Lindelöf) space. But $(\mathbb{R}, \mathcal{D})$ is not compact, so $(\mathcal{U}, Ind(\mathcal{D}), \mathbb{R})$ is not soft compact.
(2) There is a soft compact space which is not a soft second-countable. Consider any compact $T_1 - space$ (X, τ) with more that $2^{\aleph_\circ}$ points. Then (X, τ) is not second countable. Consider $(\mathcal{U}, Ind(\tau), X)$ where the universe $\mathcal{U}$ is any nonempty set. Then $(\mathcal{U}, Ind(\tau), X)$ is soft compact but not soft second countable.

Theorem 2.8. *Consider $SS(\mathcal{U})_{\mathcal{P}}$ and $SS(V)_{\mathcal{P}'}$ and let $u : \mathcal{U} \to V$ and $p : \mathcal{P} \to \mathcal{P}'$ be mappings.*

(1) $f_{pu}^{-1}(Ind(B), \mathcal{P}') = (Ind(p^{-1}(B)), \mathcal{P})$ for all $B \subset \mathcal{P}'$.
(2) If u is onto, then $f_{pu}(Ind(A), \mathcal{P}) = (Ind(p(A)), \mathcal{P}')$ for all $A \subset \mathcal{P}$.

Proof. (1)

$$f_{pu}^{-1}(Ind(B'), \mathcal{P}')(e) = u^{-1}(Ind(B')(p(e))) = \begin{cases} u^{-1}(\mathcal{U}') & \text{if } p(e) \in B' \\ \emptyset & \text{if } p(e) \notin B' \end{cases}$$

$$= \begin{cases} \mathcal{U} & \text{if } e \in p^{-1}(B') \\ \emptyset & \text{if } e \notin p^{-1}(B') \end{cases}$$

$$= (Ind(p^{-1}(B)), \mathcal{P})(e)$$

(2) First we note that $e \in p^{-1}(e') \cap A \; iff \; p(e) = e'$ and $e' \in p(A)$.

$$f_{pu}(Ind(A), \mathcal{P})(e') = \begin{cases} \bigcup_{e \in p^{-1}(e') \cap A} u(Ind(A)(e)) & \text{if } p^{-1}(e') \cap A \neq \emptyset \\ \emptyset & \text{if } p^{-1}(e') \cap A = \emptyset \end{cases}$$

$$= \begin{cases} \bigcup_{e \in p^{-1}(e') \cap A} u(\mathcal{U}) & \text{if } e' \in p(A) \\ \emptyset & \text{if } e' \notin p(A) \end{cases}$$

$$= \begin{cases} \mathcal{U}' & \text{if } e' \in p(A) \\ \emptyset & \text{if } e' \notin p(A) \end{cases}$$

$$= (Ind(p(A), \mathcal{P}')$$

□

Theorem 2.9. *Consider $SS(\mathcal{U})_{\mathcal{P}}$ and $SS(V)_{\mathcal{P}'}$ and let $u : \mathcal{U} \to V$ and $p : \mathcal{P} \to \mathcal{P}'$ be mappings. Suppose that τ and τ' are two topologies on $\mathcal{P}$ and $\mathcal{P}'$ respectively, then we have:*

(1) $f_{pu} : (\mathcal{U}, Ind(\tau), \mathcal{P}) \to (V, Ind(\tau'), \mathcal{P}')$ is soft continuous iff $p : (\mathcal{P}, \tau) \to (\mathcal{P}', \tau)$ is continuous.
(2) If u is onto, then $f_{pu} : (\mathcal{U}, Ind(\tau), \mathcal{P}) \to (V, Ind(\tau'), \mathcal{P}')$ is soft open (closed) iff $p : (\mathcal{P}, \tau) \to (\mathcal{P}', \tau)$ is open (closed).
(3) If u is one-to-one and onto, then $f_{pu} : (\mathcal{U}, Ind(\tau), \mathcal{P}) \to (V, Ind(\tau'), \mathcal{P}')$ is a soft homeomorphism iff $p : (\mathcal{P}, \tau) \to (\mathcal{P}', \tau)$ is a homeomorphism.

Proof. Apply the above theorem. □

We can get a similar theorem for soft quasi-continuity. First we define soft quasi-continuity and study some of its properties.

Definition 2.10. Consider $SS(\mathcal{U})_{\mathcal{P}}$ and $SS(V)_{\mathcal{P}'}$ and let $u : \mathcal{U} \to V$ and $p : \mathcal{P} \to \mathcal{P}'$ be mappings. For any two soft topologies $\mathcal{T}$ and $\mathcal{T}'$ on $SS(\mathcal{U})_{\mathcal{P}}$ and $SS(V)_{\mathcal{P}'}$ respectively the mapping $f_{pu} : (\mathcal{U}, \mathcal{T}, \mathcal{P}) \to (V, \mathcal{T}', \mathcal{P}')$ is called *soft quasi-continuous* iff for all soft point $e_F \in \mathcal{U}_{\mathcal{P}}$ and every soft open sets $(F, \mathcal{P}) \in \mathcal{T}$ and $(G, \mathcal{P}') \in \mathcal{T}'$ such that $e_F \widetilde{\in} (F, \mathcal{P})$ and $f_{pu}(e_F) \widetilde{\in} (G, \mathcal{P}')$ there exist a nonempty soft open (in $\mathcal{T}$)) set $(K, \mathcal{P}) \widetilde{\subset} (F, \mathcal{P})$ with $f_{pu}(K, \mathcal{P}) \widetilde{\subset} (G, \mathcal{P}')$.

The following is obvious!

Theorem 2.11. *Consider $SS(\mathcal{U})_{\mathcal{P}}$ and $SS(V)_{\mathcal{P}'}$. Let $u : \mathcal{U} \to V$ and $p : \mathcal{P} \to \mathcal{P}'$ be mappings, and consider the soft topologies $\mathcal{T}$ and $\mathcal{T}'$ on $SS(\mathcal{U})_{\mathcal{P}}$ and $SS(V)_{\mathcal{P}'}$ respectively. If f_{pu} is soft continuous, then it is soft quasi-continuous.*

The following theorem will be used later to prove the faulty of the converse.

Theorem 2.12. *Consider $SS(\mathcal{U})_{\mathcal{P}}$ and $SS(V)_{\mathcal{P}'}$ and let $u : \mathcal{U} \to V$ and $p : \mathcal{P} \to \mathcal{P}'$ be mappings. Then $f_{pu} : (\mathcal{U}, Ind(\tau), \mathcal{P}) \to (V, Ind(\tau'), \mathcal{P}')$ is soft quasi continuous iff $p : (\mathcal{P}, \tau) \to (\mathcal{P}', \tau)$ is quasi continuous where τ and τ' are topologies on $\mathcal{P}$ and $\mathcal{P}'$ respectively.*

Proof. Let f_{pu} be a soft quasi continuous, $e \in A \in \tau$ and $p(e) \in B \in \tau'$. Consider the soft open sets $(Ind(A), \mathcal{P})$, $(Ind(B), \mathcal{P}')$ and e_F in $SS(\mathcal{U})_{\mathcal{P}}$. Since $e \in A$, we have $e_F \widetilde{\in} (Ind(A), \mathcal{U})$. Since $f_{pu}(e_F)$ is a soft point and $p(e) \in B$, we have $f_{pu}(e_F) \widetilde{\in} (Ind(B), \mathcal{P}')$. Since f_{pu} is soft quasi continuous, there exists a soft open set $(Ind(C), \mathcal{P}) \widetilde{\subset} (Ind(A), \mathcal{P})$ such that $f_{pu}(Ind(C), \mathcal{P}) = (Ind(P(C)), \mathcal{P}')$ $\widetilde{\subset} (Ind(B), \mathcal{P}')$, from which we have $C \subset A$ and $p(C) \subset B$, that is p is quasi continuous. Conversely; Suppose that p is quasi continuous and consider the soft point e_F from $SS(\mathcal{U})_{\mathcal{P}}$ such that $e_F \widetilde{\in} (Ind(A), \mathcal{P})$ and $f_{pu}(e_F) \widetilde{\in} (Ind(B), \mathcal{P}')$ where $(Ind(A), \mathcal{P}) \widetilde{\in} Ind(\tau)$ and $(Ind(B)), \mathcal{P}') \widetilde{\in} Ind(\tau')$. It is clear that $e \in A \in \tau$ and $p(e) \in B \in \tau'$. But p is quasi continuous, so there exist an open set $C \in \tau$ such that $C \subset A$ and $p(C) \subset B$. Which implies that $(Ind(C), \mathcal{P})$ is soft open subset of $(Ind(A), \mathcal{P})$ and $f_{pu}(Ind(C), \mathcal{P}) = (Ind(p(C), \mathcal{P}')) \widetilde{\subset} (Ind(B), \mathcal{P}')$. That is f_{pu} is soft quasi continuous. □

The following example show that soft quasi continuity does not imply soft continuity.

Example 2.13. Let the universes be $\mathcal{U} = \mathcal{U}'$ be nonempty and the sets of parameters be $\mathcal{P} = \mathcal{P}' = \mathbb{R}$ equipped with the usual topology $\tau = \tau' = \mathcal{U}$. Let $u : \mathcal{U} \to \mathcal{U}'$ be the identity(can be any function) and define $p : (\mathcal{P}, \tau) \to (\mathcal{P}', \tau)$ as follows

$$p(e) = \begin{cases} 1 & \text{if } e \geq 0 \\ -1 & \text{if } e < 0 \end{cases}$$

We know that p is quasi continuous but not continuous, so f_{pu} is soft quasi-continuous and not soft continuous.

3. Soft Separation Axioms

In the definition of $Ind(\tau)$ there is no restrictions on the universe $\mathcal{U}$ it is just any nonempty set. Here we will put a restriction on it specially when we discuss the law soft separation axioms. Before this we have the following about soft regularity and soft normality.

Theorem 3.1. *Let $\mathcal{U}$ be the initial universe, $\mathcal{P}$ be a fixed set of parameters. Let τ be a topology on $\mathcal{P}$, and let $Ind(\tau)$ be the indicator topology generated on $SS(\mathcal{U})$ by τ.Then*

(1) $Ind(\tau)$ is soft regular iff τ is regular.
(2) $Ind(\tau)$ is soft normal iff τ is normal.

Proof. (1) Suppose that $Ind(\tau)$ is soft regular. Let $e \in \mathcal{P}$ and C be closed in $\mathcal{P}$ with $e \notin C$. Consider any soft point e_F and $(Ind(C), \mathcal{P})$ which is soft closed. Since $e \notin C$ we have $e_F \widetilde{\notin} (Ind(C), \mathcal{P})$, but $Ind(\tau)$ is soft regular, so there is disjoint soft open sets $(Ind(\mathcal{U}), \mathcal{P})$ and $(Ind(V), \mathcal{P})$ with $e \widetilde{\in} (Ind(\mathcal{U}), \mathcal{P})$ and $C \widetilde{\subset} (Ind(V), \mathcal{P})$. It is clear that $\mathcal{U}$ and V are disjoint open sets in τ and $e \in \mathcal{U}$ and $C \subset V$ which implies regularity ofτ.
Conversely, suppose that τ is regular and consider a soft point $e_F \notin (Ind(C), \mathcal{P})$ where C is closed in τ. This means $e \notin C$. Since τ is regular, there are two disjoint and open sets U and V such that $e \in U$ and $C \subset V$. It is clear that $(Ind(U), \mathcal{P})$ and $(Ind(V), \mathcal{P})$ are disjoint and containing e_F and $(Ind(C), \mathcal{P})$ respectively, so $Ind(\tau)$ is soft regular.
(2) Similar to the discussion above.

□

Note that the indicator soft topology is not a discrete topology, this is can be proved easily using the above theorem, since for any none-regular topology τ on the set of parameters, we have $Ind(\tau)$ is not soft regular, so it is not discrete.

Now about low separation axioms. Since we are looking for a soft topology satisfying the same corresponding soft properties as a given topological space, we can assuming some restrictions on the universe. Assume that the universe is a singleton $\mathcal{U} = \{x\}$, in this case the indicator topology $Ind(\tau)$ will be called *Embedding* of τ in $SS(\mathcal{U})_{\mathcal{P}}$ and will be denoted by $Emb(\tau)$. Note that every result we have proved for $Ind(\tau)$ is true if we replace $Ind(\tau)$ by $Emb(\tau)$. If the universe is singleton we have the following obvious result about distinct soft points.

Theorem 3.2. *If the universe $\mathcal{U} = \{x\}$ is a single point and e_F and e'_G are two soft point where $e, e' \in \mathcal{P}$, then we have $e_F \neq e'_G$ iff $e \neq e'$.*

The importance of this simple result is combining distinct soft point with distinct point in the set of parameters whenever the universe is a single point. So we will not face a situation where we have two distinct soft point $e_F \neq e'_G$ and $e = e'$.

Theorem 3.3. *Consider the universe $\mathcal{U}$, and the parameters $\mathcal{P}$. Let τ be a topology on $\mathcal{P}$, and let $Emb(\tau)$ be the Embedding topology generated on $SS(\mathcal{U})$ by τ.Then*

(1) $Emb(\tau)$ is soft T_0 iff τ is T_0.
(2) $Emb(\tau)$ is soft T_1 iff τ is T_1.
(3) $Emb(\tau)$ is soft T_2 iff τ is T_2.
(4) $Emb(\tau)$ is soft T_3 iff τ is T_3.
(5) $Emb(\tau)$ is soft T_4 iff τ is T_4.

Proof. We will proof (3) and the rest can be proved in the same manner.
(3) Suppose that $Emb(\tau)$ is soft T_2. Let $e \neq e'$ in $\mathcal{P}$. Then e_F and e'_G are distinct soft sets. So there are two disjoint soft open sets $(Ind(U), \mathcal{P})$ and $(Ind(V), \mathcal{P})$ such that $e_F \widetilde{\in} (Ind(U), \mathcal{P})$ and $e'_G \widetilde{\in} (Ind(V), \mathcal{P})$. It is clear that U and V are disjoint and open

sets in τ and $e \in U$ and $e' \in V$ which implies τ is T_2.
Conversely, suppose that τ is T_2 and consider two distinct soft point $e_F, e'_G \widetilde{\in} SS(\mathcal{U})_{\mathcal{P}}$. Since $\mathcal{U}$ is a single point, we have $e \neq e'$. And since τ is T_2, there exists two disjoint and open sets $\mathcal{U}$ and V in τ such that $e \in U$ and $e' \subset V$. It is clear that $(Ind(\mathcal{U}), \mathcal{P})$ and $(Ind(V), \mathcal{P})$ are soft open and disjoint containing e_F and e'_G, so $Emb(\tau)$ is soft T_2. □

Remark : In the proof of the above theorem being $\mathcal{U}$ a singleton is used in the proof of one direction: If τ is T_i , then $Emb(\tau)$ is soft T_i where $i = 1, 2, 3, 4$; the other direction can be proved for any universe $\mathcal{U}$ so it can be proved for $Ind(\tau)$.

Example 3.4.
(1) Let $(\mathcal{P}, \tau)$ be any $T_0 - space$ which is not T_1, and the universe $\mathcal{U}$ be a singleton. Then the soft topology $Emb(\tau)$ is soft T_0 but not soft T_1 .
(2) Let $(\mathcal{P}, \tau)$ be any $T_1 - space$ which is not T_2, and the universe $\mathcal{U}$ be a singleton. Then the soft topology $Emb(\tau)$ is soft T_1 but not soft T_2 .
(3) Let $(\mathcal{P}, \tau)$ be any $T_2 - space$ which is not T_3, and the universe $\mathcal{U}$ be a singleton. Then the soft topology $Emb(\tau)$ is soft T_2 but not soft T_3 .
(4) Let $(\mathcal{P}, \tau)$ be any $T_3 - space$ which is not T_4, and the universe $\mathcal{U}$ be a singleton. Then the soft topology $Emb(\tau)$ is soft T_3 but not soft T_4.

Now, we construct an example to shows that soft T_0 is not preserved under soft open, closed and onto soft mappings.

Example 3.5. Let $\mathcal{P}$ be the set of all integers equipped with the topology $\tau = \{A_n; n \in \mathcal{P}\}$ where $A_n = \{n, n+1, n+2, ...\}$. Then $(\mathcal{P}, \tau)$ is a $T_0 - sapce$. And let $\mathcal{P}' = \{0, 1\}$ equipped with the indiscrete topology τ'. It is clear that $(\mathcal{P}', \tau')$ is not a $T_0 - sapce$. Define $p : (\mathcal{P}, \tau) \to (\mathcal{P}', \tau')$ as follows $p(x) = 0$ if x is odd and $p(x) = 1$ if x is even. It is clear that p is closed-and-open mapping from $\mathcal{P}$ onto $\mathcal{P}'$. Now let our universes be $\mathcal{U} = \mathcal{U}' = \{a\}$ be singletons and define $u : \mathcal{U} \to \mathcal{U}'$ by $u(a) = a$. Then from the above theorems and Theorem2.9 we have the following:
(1) $(\mathcal{U}, Emb(\tau), \mathcal{P})$ is a soft $T_0 - sapce$.
(2) $(\mathcal{U}', Emb(\tau'), \mathcal{P}'))$ is not a soft $T_0 - sapce$.
(3) $f_{pu} : SS(\mathcal{U})_{\mathcal{P}} \to SS(\mathcal{U}')_{\mathcal{P}'}$ is a soft open-and-closed onto mapping.
So soft T_0 is not preserved under soft open-and-closed onto mappings.

The following example shows none of the soft separation axioms is preserved under soft open and onto mappings

Example 3.6. Let $\mathcal{P}$ be the set of real numbers equipped with the usual topology τ. Then it is clear that $(\mathcal{P}, \tau)$ is $T_4 - sapce$. And let $\mathcal{P}' = \{0, 1\}$ equipped with the indiscrete topology τ'. It is clear that $(\mathcal{P}', \tau')$ is not a $T_0 - sapce$. Define $p : (\mathcal{P}, \tau) \to (\mathcal{P}', \tau')$ as follows $p(x) = 0$ if x is rational and $p(x) = 1$ if x is irrational. It is clear that p is open and onto mapping from $\mathcal{P}$ onto $\mathcal{P}'$. Now let our universes be $\mathcal{U} = \mathcal{U}' = \{a\}$ be singletons and define $u : \mathcal{U} \to \mathcal{U}'$ by $u(a) = a$. Then from the above theorems and Theorem2.9 we have the following:
(1) $(\mathcal{U}, Emb(\tau), \mathcal{P})$ is a soft $T_4 - sapce$.
(2) $(\mathcal{U}', Emb(\tau'), \mathcal{P}'))$ is not a soft $T_0 - sapce$.
(3) $f_{pu} : SS(\mathcal{U})_{\mathcal{P}} \to SS(\mathcal{U}')_{\mathcal{P}'}$ is a soft open and onto mapping.

So soft T_0,T_1,T_2,T_3 and T_4 are not preserved under soft open and onto mappings.

4. Applications and further studies

This paper introduce a new concepts about soft sets called indicator soft set can be constructed using any subset of parameters, this new concept used to generate a soft topology for any given topology on the set of parameters; these tow topologies have the same corresponding topological properties such as separation axioms, different kinds of compactness and gneralization, continuity, ... etc. We can apply the properties of indicator topology to get counter examples for soft topological spaces, since it is not an easy task to construct counter examples for advanced notations in soft topology, such as an example of a soft $T_4 - space$ that is not a $T_3 - space$, but using indicator soft topology we just use the corresponding counterexample in general topology and construct the indicator topology for it and the resulting one is the desired counter example, and this is can be done for many notation in soft topology. Our further studies will be concentrated on applying the indicator soft set for more notation in topology and other disciplines, and we need to study the existing definitions of soft points where there are two different definitions for soft points; one of them considers soft points as soft sets and the other as a point in the set of parameters, so we need to study the difference between these two definitions and which one is more functional.

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