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CHARACTERIZATIONS OF J, J*, J**-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract. In this paper, J,J* and J** - closed sets are introduced in topological spaces. The relationships of these closed sets are analysed with some existing g - closed sets in the literature. In spaces like $T_{1/2}$, almost weakly Hausdorff, T_δ, semi - regular, T_b, αT_b, partition space, R_1, these closed sets are characterized.

1. Introduction

In 1937, Stone [32] introduced regular open sets and used it to define the semiregularization of a topological space. In 1968, Velicko [36] proposed δ - open sets which are stronger than open sets. Levine [17] has brought generalized closed sets in 1970. Dunham [10] has established a generalized closure using Levine′s generalized closed sets as Cl*. In 2016,Pious Missier [29] has introduced regular* - open sets using Cl*. In 2018, Meenakshi and Sivakamasundari [21] have introduced a class of new sets namely η^* - open sets which is placed between the classes of δ - open sets and open sets. Its basic properties are obtained and the concepts of η^* - cluster point, η^* - adherent point and a η^* - derived set are introduced and studied in the same paper. Using η^* - open sets, they have also introduced J - closed sets [20] and their properties are studied. Moreover g - closed $\rightarrow$ J - closed set $\rightarrow$ gδ - closed set [7] and δg - closed set [8] $\rightarrow$ J* - closed set $\rightarrow$ g - closed set, δg - closed set $\rightarrow$ J** - closed set $\rightarrow$ g - closed set. In this paper, J, J* and J** - closed sets are introduced in topological spaces. The relationships of these closed sets are analysed with some existing g - closed sets in the literature. In spaces like $T_{1/2}$ [17], almost weakly Hausdorff [8], T_δ [8], semi-regular, T_b [6], αT$_b$ [5], Partition space [25], R$_1$ [4] these closed sets are characterized.

Some basic definitions and results in topological spaces are needed which are given in section 2 and will be used in the sequel. Throughout this paper, (Y, ζ) will always denote a topological space.

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2. Preliminaries

Definition 2.1. Let (Y, ζ) be a topological space. If D is a non - empty subset of (Y,ζ) then the intersection of all closed sets containing D is called closure of D and is denoted by Cl(D). The union of all open sets contained in D is called interior of D and is denoted by int(D).

Definition 2.2. A subset D of a topological space (Y, ζ)is called generalized closed (briefly g - closed) [17] if Cl(D) $\subseteq$ M whenever D $\subseteq$ M and M is open in (Y,ζ).

Definition 2.3. If D is a subset of a space (Y,ζ).

(i) The generalized closure [10] of D is defined as the intersection of all g - closed sets in Y containing D and is denoted by Cl*(D).
(ii) The generalized interior [10] of D is defined as the union of all g - open sets in Y contained in D and is denoted by int*(D).

Definition 2.4. Let (Y,ζ) be a topological space. A subset D of (Y,ζ)is called

(1) regular closed set [32] if D = Cl(int(D)).
(2) semi - closed set [16] if int(Cl(D)) $\subseteq$D.
(3) α - closed set [26] if Cl(int(Cl(D))$\subseteq$D
(4) pre - closed set [19]if Cl(int(D)) $\subseteq$D
(5) semi pre - closed set [1] if int(Cl(int(D))) $\subseteq$D
(6) π - closed set [37] if it is the finite union of regular closed sets.

The complements of the above mentioned sets are called regular open, semi - open, α - open, pre - open and semi pre - open, π - open sets respectively.

The intersection of a particular class of closed sets of Y containing D is called the corresponding closure of D.

Definition 2.5. [36] A set is δ-open if it is the union of regular open sets, the complement of a δ - open set is called a δ - closed set.

Definition 2.6. Let (Y,ζ) be a topological space. A subset D of (Y,ζ) is called regular * - open (or r* - open) [29] if D = int(Cl*(D)). The complement of regular* - open set is called regular * - closed set(or r* - closed). The union of all regular* - open sets of Y contained in D is called regular* - interior and is denoted by r*int(D). The intersection of all regular* - closed sets of Y containing D is called regular* - closure is denoted by r*Cl(D).

Definition 2.7. A subset D of a topological space (Y,ζ) is called

(1) generalized semi - closed(briefly gs - closed) [2] if sCl(D)$\subseteq$ M whenever D $\subseteq$ M and M is open in (Y, ζ).
(2) generalized δ - closed (briefly gδ - closed) [7] if Cl(D)$\subseteq$ M whenever D$\subseteq$ M and M is δ - open in (Y, ζ).
(3) δg‡ - closed [7] if δCl(D) $\subseteq$ M whenever D$\subseteq$ M and M is δ - open in (Y, ζ).
(4) δ generalized - closed(briefly δg - closed) [8] if δCl(D) $\subseteq$M whenever D$\subseteq$ M and M is open in (Y, ζ).
(5) α - generalized closed(briefly αg - closed)[18] if αCl(D)$\subseteq$ M whenever D$\subseteq$M and M is open in (Y, ζ).

(6) δ - generalized star - closed (briefly δg^* - closed)[33] if δCl(D) $\subseteq$M whenever D$\subseteq$ M and M is g - open in (Y, ζ).
(7) generalized pre regular - closed(briefly gpr - closed) [11] if pCl(D) $\subseteq M$ whenever D $\subseteq$M and M is regular open in (Y, ζ).
(8) π - generalized closed(briefly πg - closed) [9] if Cl(D)$\subseteq$M whenever D$\subseteq$M and M is π - open in (Y, ζ).
(9) $\hat{g}$ - closed [35] if Cl(D) $\subseteq$M whenever D$\subseteq$M and M is semi-open in (Y, ζ).
(10) $^{\#}$gs - closed[34] if sCl(D) $\subseteq$M whenever D$\subseteq$M and M is $^*g$ - open in (Y, ζ).
(11) π - generalized semi - closed(briefly πgs - closed) [3] if sCl(D) $\subseteq$M whenever D$\subseteq$M and M is π - open in (Y, ζ).
(12) π - generalized pre - closed(briefly πgp - closed) [28] if pCl(D) $\subseteq$M whenever D$\subseteq$M and M is π - open in (Y, ζ).
(13) $^*g$ - closed [12] if Cl(D) $\subseteq$M whenever D$\subseteq$M and M is $\hat{g}$ - open in (Y, ζ).
(14) regular weakly generalized - closed (briefly rwg - closed) [23] if Cl(int(D)) $\subseteq$M whenever D$\subseteq$M and M is regular open in (Y, ζ).
(15) π - generalized α - closed(briefly πgα - closed) [13] if α Cl(D) $\subseteq$M whenever D$\subseteq$ M and M is π - open in (Y, ζ).
(16) π - generalized semi pre - closed(briefly πgsp - closed)[31] if spCl(D)$\subseteq$M and M is π - open in (Y, ζ).
(17) generalized semi pre regular - closed(briefly gspr - closed) [24] if spCl(D) $\subseteq$ M whenever D$\subseteq$M and M is regular open in (Y, ζ).
(18) g*s - closed [30] if sCl(D)$\subseteq$M whenever D$\subseteq$M and M is gs - open in (Y, ζ).
(19) g* - closed [22] if Cl(D) $\subseteq$M whenever D$\subseteq$M and M is g - open in (Y, ζ).
(20) regular generalized closed (briefly rg - closed) [27] if Cl(D)$\subseteq$M whenever D$\subseteq$ M and M is regular open in (Y, ζ).

The complements of the above mentioned sets are called their respective open sets.

Remark 2.8. A topological space (Y, ζ) is said to be a

(1) $T_{1/2}$ - space[17]if every g - closed subset of (Y, ζ) is closed in (Y,ζ).
(2) Semi regular space [8] if $\zeta = \zeta_s$, the family ζ_s is known as semi - regularization of ζ. In a semi regular space, the family of all δ - open sets coincide with that of open sets.
(3) T_b - space [6] if every gs - closed subset of (Y, ζ) is closed in (Y, ζ).
(4) αT_b - space [5] if every αg - closed subset of (Y, ζ) is closed in (Y, ζ).
(5) T_δ - space [7] if every gδ - closed subset of (Y, ζ) is δ - closed in (Y, ζ).
(6) A topological space (Y, ζ) is called R_1 - space[4] if every two different points with distinct closures have disjoint neighbourhoods.
(7) A partition space [25] is a topological space where every open set is closed.
(8) Almost weakly Hausdorff space [8] if its semi regularization is $T_{1/2}$.

3. η^* - Open Sets

Definition 3.1. A subset D of a topological space (Y, ζ) is called η^* - open [21] set if it is a union of regular* - open sets. The complement of a η^* - open set is called a η^* - closed set. The intersection of all η^* - closed sets of Y containing D is called as the η^* -

closure of D and is denoted by η^* - Cl(D).
We denote the set of all η^* - open sets in (Y, ζ) by η^*O(Y, ζ) or η^*O(Y).

Example 3.2. Let Y={p,q,r}, ζ={ϕ, Y, {p},{q},{p,q}}.Then r*O(Y) ={ϕ, Y,{p},{q}}, η^*O(Y) ={ϕ,Y,{p},{q},{p,q}}.

Theorem 3.3. *Every δ - open set is a η^* - open set but not conversely.*

Proof. Let D be a δ - open set. Then D is a union of regular open sets. Take x $\in$ D, then x $\in \cup$ B_μ, where B_μ is regular open. Since every regular open set is regular* - open,**Theorem 3.12.** of [29], D is a union of regular* - open sets. Thus D is a η^* - open set. Hence every δ - open is η^* - open. □

Example 3.4. Let Y={p,q,r}, $\boldsymbol{\zeta}$={$\boldsymbol{\phi}$, Y,{p}}. Then the subset {p} is a η^* - open set but it is not a δ - open set in (Y,$\boldsymbol{\zeta}$).

Theorem 3.5. *Every η^* - open set is an open set but not conversely.*

Proof. Since every r* - open set is an open set [**Theorem 3.15.** of [29]]. The proof follws as in the proof of **Theorem 3.3.**. □

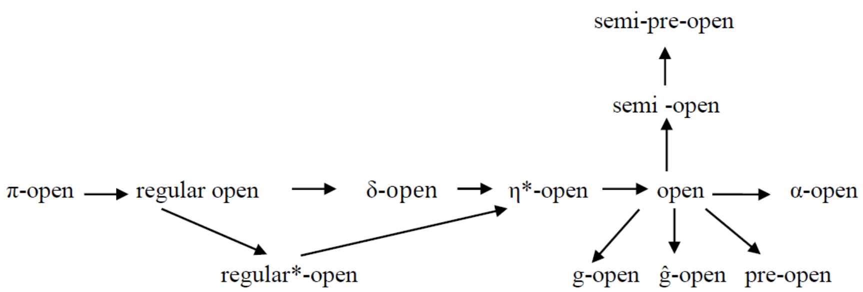
Example 3.6. Let Y={p,q,r}, ζ = {ϕ, Y, {p,q}}. Then the subset {p,q} is an open set but it is not a η^*-open set in (Y,ζ).

Theorem 3.7. *Every r* - open set is a η^*-open set but not conversely.*

Proof. Follows from **Definition 3.1.**. □

Example 3.8. Consider the **Example 3.2.**,the subset {p,q} is a η^* - open set but it is not a r* - open set in (Y,ζ).

Note 3.9. From the above Theorems we get the following implication diagram.



Theorem 3.10.
(a) *In any topological space (Y,ζ), Y and ϕ are η^* - open sets.*
(b) *Any arbitrary union of η^* - open sets is a η^* - open set.*
(c) *The finite intersection of η^* - open sets is a η^* - open set*

Definition 3.11. The Complement of a η^* - open set is called a η^* - closed set. We denote η^* - closed sets in (Y,ζ) by η^*C(Y,ζ) or η^*C(Y).

Remark 3.12. The family of η^* - closed sets is properly placed between the family of δ - closed sets and closed sets.

Proposition 3.13. In a $T_{1/2}$ - space, η^* - closed sets coincide with δ - closed sets.

Proof. In a $T_{1/2}$ - space,g - closed sets coincide with closed sets. Therefore generalized closure of a set coincides with closure of a set. Hence regular* - open sets are regular open sets in $T_{1/2}$ - spaces and η^* - closed sets coincide with δ - closed sets. □

Example 3.14. Let Y = {p,q,r}, ζ = {ϕ,Y,{p},{q},{p,q},{p,r}}.
Here ζ^c = gC(Y,ζ) = {ϕ, Y, {q},{r},{q,r},{p,r}} represents a $T_{1/2}$ - space and also η^*C(Y,ζ) = δC(Y,ζ) = {ϕ, Y,{q}{p,r}.
The converse of **Proposition 3.13.** is not true seen from the following Example.

Example 3.15. Let Y = {p,q,r}, ζ = {ϕ, Y, {p,q}}. Here η^*C(Y,ζ) = δC(Y,ζ) = {ϕ, Y} but it is not a $T_{1/2}$ - space.

Proposition 3.16. If a space is a semi - regular space, then η^* - closed sets coincide with closed sets.

Proof. It is obvious. □

Remark 3.17. In any topological space (Y, ζ)
(i) η^* -(η^* - Cl(D))=η^* - Cl(D)
(ii) η^* - Cl(D)=D iff D is η^* - closed.

4. J - closed sets

Definition 4.1. A subset D of a topological space (Y,ζ) is said to be a J - closed Set [20] if Cl(D)$\subseteq$ M whenever D $\subseteq$ M, where M is η^* - open in (Y,ζ). The class of all J - closed sets of (Y,ζ) is denoted by JC(Y,ζ).

Proposition 4.2. Every closed set is J - closed but not conversely.

Proof. Let D be a closed set and M be any η^* - open set containing D. Since D is closed, Cl(D) = D. Therefore Cl(D) = D $\subseteq$ M. Hence D is J - closed. □

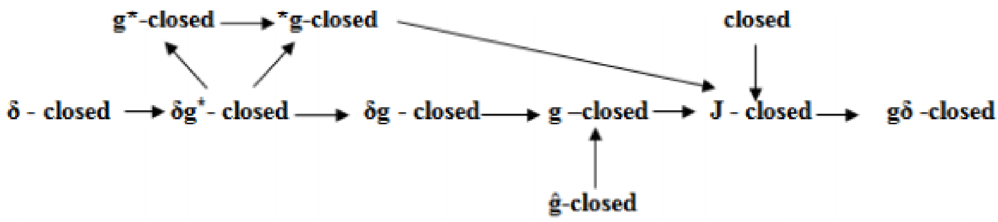
Example 4.3. Let Y = {p,q,r}, ζ = {ϕ,Y, {p},{p,q}}. η^*O(Y,ζ) = {ϕ,Y,{p}} and JC(Y,ζ) = P(Y)- {p}. The subset {q} is J - closed but not closed.

Theorem 4.4. *Every δ - closed (resp. δg^* - closed, δg - closed, g - closed , g^* - closed, $\hat{g}$ - closed, $^*g$ - closed) set is a J - closed set but not conversely.*

Example 4.5. In **Example 4.3.**, {r} is J - closed but not δ - closed. In the same Example, {q} is J - closed but not g - closed, g* - closed, $\hat{g}$ - closed and *g - closed.

Example 4.6. Let Y = {p,q,r},ζ = {ϕ, Y, {p}, {p,q} {p,r}}. Then the subset {q} is J - closed but not δg^* - closed and δg - closed in (Y,ζ).

Remark 4.7.

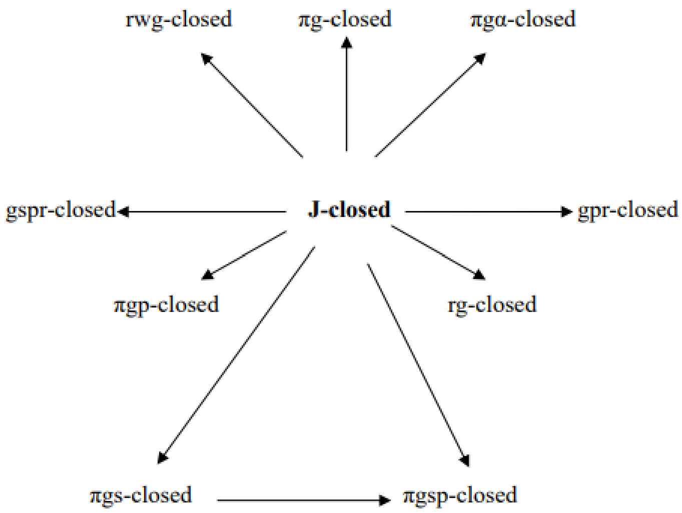


Theorem 4.8. *Every J - closed set is (resp. gδ-closed, rg-closed, gpr-closed, rwg-closed, gspr-closed, πg-closed, πgp-closed, πgsp-closed, πgs-closed, πgα-closed) set but not conversely.*

Example 4.9. In **Example 4.3.**, {p} is gδ - closed, rg - closed, πg - closed,πgp - closed, πgα - closed but not J - closed.

Example 4.10. Let Y = {p,q,r},ζ = {ϕ, Y,{p},{q},{p,q}}. Then the subset {p,q} is gpr - closed,rwg - closed and {q} is gspr - closed ,{p} is πgs - closed, πgsp - closed but not J - closed in (Y, ζ).

Remark 4.11. From the above discussions it is interesting to observe that J-closed sets is stronger than some existing g-closed sets as given below.
In the below diagram, A $\rightarrow$ B represents A implies B the reverse implication need not be true.



Remark 4.12. The following examples show that J - closed set is independent from gs - closed, #gs - closed, g*s - closed, αg - closed and $\delta g^{\ddagger}$ - closed sets.

Example 4.13. In **Example 4.10.**,the subset {p} is gs - closed, #gs - closed, g*s - closed but not J - closed in (Y,ζ).

Example 4.14. Let Y = {p,q,r},ζ = {ϕ, Y, {p,q}} The subset {p} is J - closed but not gs - closed, #gs - closed, g*s - closed in (Y,ζ).

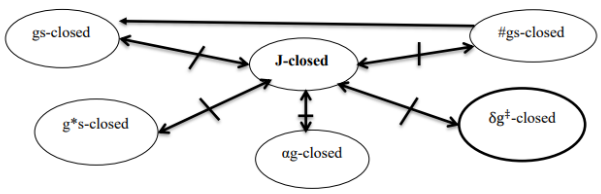
Example 4.15. Let Y = {p,q,r,s},ζ = {ϕ, Y, {p}}. The subset {p} is not J - closed but it is $\delta g^{\ddagger}$ - closed in (Y,ζ).

Example 4.16. Let Y = {p,q,r,s},ζ = {ϕ,Y,{p}, {r},{p,q},{p,r},{p,q,r},{p,r,s}} The subset {q} is J - closed but not $\delta g^{\ddagger}$ - closed in (Y,ζ).

Example 4.17. In **Example 4.3.**,the subset {p,q} is J - closed but not αg - closed in (Y,ζ).

Example 4.18. In **Example 4.3.**,The subset {q} is αg-closed but it is not J - closed in (Y,ζ).

Remark 4.19. From the above discussions it can be seen that J - closed sets is isolated from some existing g - closed sets.(In this diagram, A $\nleftrightarrow$B represents A and B are isolated).



5. J* - closed sets

Definition 5.1. Let D $\subseteq$ Y. A subset D of (Y,ζ) is known as J* - closed set if η^* - Cl(D)$\subseteq$M whenever D$\subseteq$M, M $\in$ ζ.

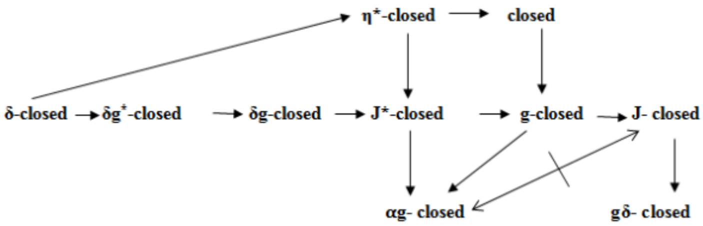
Lemma 5.2. *Every η^* - closed set is a J* - closed set.*

Proof. Let D be η^* - closed. Assume that M $\in$ ζ containing D. Since D is η^* - closed, η^* - Cl(D) = D $\subseteq$ M. Therefore D becomes J* - closed. $\square$

Theorem 5.3. *Every δ - closed (resp. δg^* - closed, δg - closed) set is a J* - closed but not conversely.*

Theorem 5.4. *Every J* - closed set is (resp. g - closed, J - closed, αg - closed, $g\delta$ - closed, rg - closed, gpr - closed, rwg - closed, gspr - closed, πg - closed, πgp - closed, πgsp - closed, πgs - closed, $\pi g\alpha$ - closed, gs - closed, gsp - closed) set but the converses may not hold good seen from following Examples.*

Remark 5.5.



Example 5.6. Consider Y = {p,q,r}, ζ={ϕ, Y, {p}}. Analysing we get

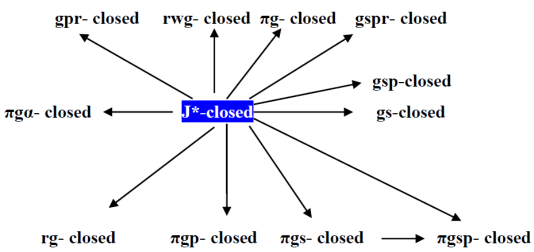
(i) {q} $\subseteq$ Y happens to be J* - closed not satisfying the condition for δ - closed
(i) {q,r} $\subseteq$ Y happens to be J *- closed not satisfying the condition for δg* - closed.

Example 5.7. Consider Y = {p,q,r}, ζ = {ϕ, Y, {r,p}, {p},{p,q}}. Analysing we get {q} $\subseteq$ Y happens to be g - closed not satisfying the condition for J* - closed.

Example 5.8. Consider Y = {p,q,r}, ζ = {ϕ, Y, {p,q}}. Analysing we get {q} $\subseteq$ Y happens to be J - closed not satisfying the condition for J* - closed .

Example 5.9. In **Example 4.3.**,analysing we get {q} $\subseteq$ Y happens to be αg - closed, πgp - closed, gsp - closed, gs - closed and in **Example 4.10.**,{q} $\subseteq$ Y happens to be gspr-closed, πgs - closed and {p} $\subseteq$ Y is πgsp - closed , {p} $\subseteq$ Y happens to be gδ-closed, rg-closed, πg - closed, πgsp - closed, and {p,q} $\subseteq$ Y happens to be πgα - closed, rwg - closed,{p,r} $\subseteq$ Y happens to be gpr - closed not satisfying the condition for J* - closed.

Remark 5.10. From all the comparisons carried out; it is interesting to observe that J* - closed sets is dependent with some existing generalized - closed sets.



Example 5.11. The following examples explain that J* - closed set is isolated of closed,#gs - closed and g*s - closed sets.

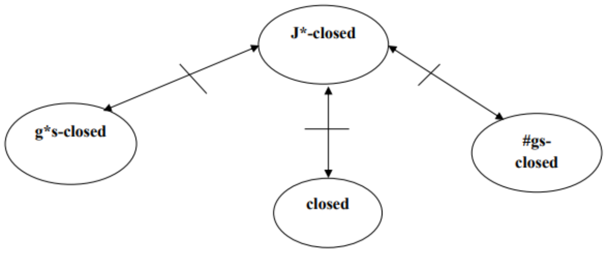
Example 5.12. In **Example 4.3.**, analysing we get {p,r} $\subseteq$ Y happens to be J* - closed but neither g*s - closed nor closed.

Example 5.13. In **Example 4.6.**, analysing we get {q} $\subseteq$ Y happens to be closed not satisfying the condition for J* - closed .

Example 5.14. In **Example 4.10.**, analysing we get {p} $\subseteq$ Y happens to be not satisfying the condition for J* - closed but it is #gs - closed and g*s - closed.

Example 5.15. In **Example 4.15.**, Analysing we get {p,q,r} $\subseteq$ Y happens to be J* - closed not satisfying the condition for #gs - closed.

Remark 5.16. From the above discussions it can be seen that J* - closed sets are isolated with some existing g - closed sets.



6. J** - closed sets

Definition 6.1. Let D $\subseteq$ Y. A subset D of (Y,ζ) is known as J** - closed set if η^* - Cl(D)$\subseteq$ M whenever D $\subseteq$ M, where M is η^* - open in (Y,ζ). We represent the family of all J** - closed sets of (Y,ζ) by J**C(Y,ζ).

Proposition 6.2. Every η^* - closed set is J** - closed but not conversely.

Proof. Let D be a η^* - closed set and M be any η^* - open set containing D. Since D is η^* - closed,η^* - Cl(D) = D (by **Remark 3.17.(ii)**). Therefore η^* - C l (D) =D $\subseteq$ M. Hence D is J** - closed. □

Example 6.3. In **Example 4.3.**,η^*O(Y,ζ) = {ϕ,Y,{p}}, The subset {q} is J** - closed but not η^* - closed.

Theorem 6.4. *Every δ - closed (resp. δg^* - closed, δg - closed, J^* - closed) set is a J^{**} - closed set but not conversely.*

Theorem 6.5. *Every J^{**} - closed set is (resp. J - closed , $g\delta$ - closed, gpr - closed, rwg - closed, gspr - closed, πg - closed, πgp-closed, πgsp - closed, πgs - closed, $\pi g\alpha$ - closed) set but the converses may not hold good seen from following Examples.*

Example 6.6. In **Example 4.3.**, analysing we get {r} $\subseteq$ Y happens to be J** - closed not satisfying the condition for δ - closed,{p} is πg - closed, πgp - closed , πgα - closed but not J** - closed.

Example 6.7. In **Example 4.6.**, analysing we get {q} $\subseteq$ Y happens to be J** - closed but not δg* - closed, δg - closed and in **Example 4.9.** {p} is πgsp - closed but not J** - closed.

Example 6.8. In **Example 4.16.**, analysing we get $\{q\} \subseteq$ Y happens to be J - closed but not J** - closed and $\{r\}$ is J** - closed but not $\delta g^{\ddagger}$ - closed.

Example 6.9. Let Y = { p,q,r,s },ζ = { ϕ ,Y,{ p }, { q },{ p,q },{ p,q,r }, { p,q,s } }, η^{*}O(Y,ζ) = { ϕ ,Y,{ p },{ q },{ p,q } }. The subset { q,r } is J** - closed but not J* - closed.

Example 6.10. Consider Y = {p,q,r,s}, ζ={ ϕ, Y, {p} }, η^{*}O(Y,ζ) = ζ. The subset {p} is g δ - closed but not J** - closed.

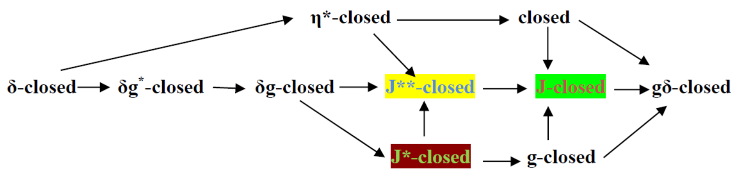
Example 6.11. In **Example 4.9.**, Analysing we get $\{p,q\} \subseteq$ Y happens to be gpr - closed, rwg - closed and {q} happens to be gspr - closed,{p} happens to be πgs - closed not satisfying the condition for J** - closed.

Example 6.12. In the above **Example 4.9.**, the subset {p} is gs-closed, #gs - closed, g*s - closed but not J** - closed.

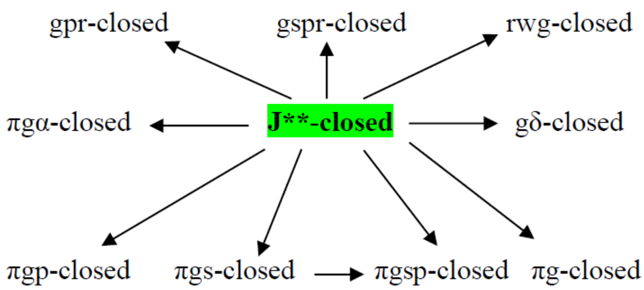
Example 6.13. Let Y = {p,q,r},ζ = {ϕ, Y,{p,q}} The subset {p} is J** - closed but not gs - closed, #gs - closed, g*s - closed.

Example 6.14. Let Y = {p,q,r,s}, ζ = {ϕ, Y, {p}}. The subset {p} is not J** - closed but it is $\delta g^{\ddagger}$ - closed.

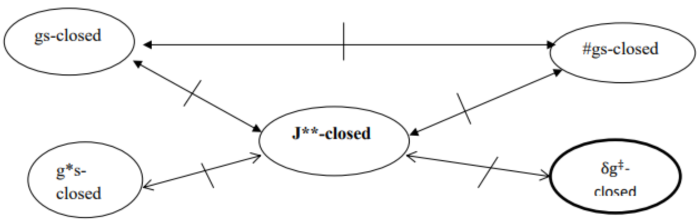
Remark 6.15.



Remark 6.16. From the above discussions it is interesting to observe that J** - closed sets are related with some existing g-closed sets in the following manner.



Remark 6.17. From the above discussions it can be seen that J** - closed sets are isolated from the following g - closed sets.



7. Characterizations of J - closed sets

Theorem 7.1. *In a $T_{1/2}$ - space for a subset D the following are equivalent:*
(i) *D is J - closed.*
(ii) *D is gδ - closed.*

Proof. $(i) \Rightarrow (ii)$ Obvious from **Theroem 4.8.**
$(ii) \Rightarrow (i)$Let D be gδ-closed, D$\subseteq$ M where M is η^* - open. In a $T_{1/2}$ - space, η^* - open sets coincide with δ - open sets (By **Proposition 3.13.**). Since D is gδ - closed, Cl(D)$\subseteq$ M $\Rightarrow$ D is J - closed. □

Example 7.2. Consider Y = {p,q,r},ζ = {ϕ, Y,{p},{q},{p,q}}. Here JC(Y,ζ) = gδC(Y,ζ) = {ϕ, Y,{r},{p,r},{q,r}} and (Y, ζ) is a $T_{1/2}$-space.

Theorem 7.3. *Let D be a subset of a semi - regular space (Y,ζ), then the following are equivalent:*
(a) *D is δg - closed.*
(b) *D is g - closed.*
(c) *D is J - closed.*
(d) *D is gδ - closed.*

Proof. $(a) \Rightarrow (b), (b) \Rightarrow (c) and (c) \Rightarrow (d)$ are true for any subset D by **Theorem 3.1.(iii)** of [8], **Theorem 4.4.** and **Theorem 4.8.** In a semi regular space, δ - closed sets coincide with closed sets. Hence $(d) \Rightarrow (a)$.

□

Corollary 7.4. Let D be a subset of a $T_{1/2}$ - space and a semi - regular space (Y,ζ), then D is J - closed if and only if D is closed.

Proof. The result follows from (b) iff (c) of **Theorem 7.3.** and the fact that in a $T_{1/2}$ - space, g - closed sets coincide with closed sets. □

Remark 7.5. In general the concepts of J - closed sets and $\delta g^{\ddagger}$ - closed sets are not equivalent. This concept is characterized in following two ways.

Corollary 7.6. J-closed sets are equivalent to $\delta g^{\ddagger}$ - closed sets in semi-regular spaces.

Proof. By **Theorem 7.3.**, J-closed sets are equivalent to δg-closed sets and gδ - closed sets. But δg - closed$\rightarrow$ $\delta g^{\ddagger}$ - closed$\rightarrow$ gδ - closed (By [7]). Hence $\delta g^{\ddagger}$ - closed sets equivalent to J - closed sets also.

But for a compact subset D of a space which is $T_{1/2}$ and R_1, the concepts of J - closed sets and $\delta g^{\ddagger}$ - closed sets coincide. This can be seen in the following Theorem. □

Theorem 7.7. *For a compact subset of a R_1 - topological space (Y,ζ), every J - closed set $\delta g^{\ddagger}$ - closed set. The converse is true if (Y,ζ) is $T_{1/2}$.*

Proof. Let D be J - closed and D be a compact subset of R_1 - topological space (Y,ζ). Let D $\subseteq$ M, where M is δ - open. By **Theorem 3.3.**, every δ - open set is η^*-open —–(1). In R_1 - spaces the concepts of closure and δ - closure coincide for compact sets (By **Theorem 3.6.** of [15]). Hence for a compact subset D,δCl(D) = Cl(D) —– (2). From (1) and (2) and since D is J - closed, we get D is a $\delta g^{\ddagger}$ - closed set.
Conversely let (Y,ζ) be $T_{1/2}$ and D be $\delta g^{\ddagger}$ - closed. Let D$\subseteq$ M, where M is η^* - open. By **Proposition 3.13.**, η^* - open sets coincide with δ - open sets in $T_{1/2}$ - spaces. By assumption, δCl(D) $\subseteq$ M which implies Cl(D) $\subseteq$ δCl(D) $\subseteq$ M. Then D becomes J - closed. □

Corollary 7.8. In Hausdroff spaces, also a finite set is J - closed if and only if it is $\delta g^{\ddagger}$ - closed.

In T_δ - spaces the equivalence in **Theorem 7.3.** becomes true. It can be seen from the following Proposition.

Proposition 7.9. If a topological space (Y,ζ) is a T_δ - space, then the following conditions are equivalent:
(a) D is δg - closed.
(b) D is g - closed.
(c) D is J - closed.
(d) D is gδ - closed.

Proof. (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c) and (c) $\Rightarrow$ (d) are true for any subset D by **Theorem 3.1.(iii)** of [8], **Theorem 4.4.** and **Theorem 4.8.** In T_δ - space, a gδ - closed set is every δ - closed set and by **Theorem 3.1.(i)** of [7], a δ - closed set is a δg - closed set. Hence $(d) \Rightarrow (a)$. □

Theorem 7.10. *The following conditions (a) and (b) hold good.*

(a) *For a topological space (Y,ζ),if Y is a partition space, then every subset of Y is J - closed.*
(b) *The converse is true if Y is a $T_{1/2}$ - space and a semi - regular space.*

Proof.
(a) Let D be an arbitrary subset of Y and M be a η^* - open set containing D. Since η^* - open is open and Y is a Partition space in which every open set is closed, M is closed. Thus Cl(D) $\subseteq$ Cl(M) = M. Hence every subset of (Y,ζ) is J - closed.

(b) Let D be an open set in (Y,ζ). By criteria, every subset of Y is J - closed. If (Y,ζ) is $T_{1/2}$ and semi regular, then J - closed sets and closed sets coincide [By **Corollary 7.4.**]. Hence D is closed in (Y,ζ). Thus (Y,ζ) is a partition space. □

Note 7.11. The converse of **Theorem 7.10.(a)** is not true which is seen in Example.

Example 7.12. Let Y={p,q,r}, ζ={ϕ, Y, {p,q}}. Here JC(Y,ζ) = P(Y). In this space every subset is J - closed but it is not a partition space.

8. Characterizations of J* - closed sets

Proposition 8.1. In a $T_{1/2}$ - space, J* - closed sets coincide with δg - closed sets.

Proof. By **Theorem 5.3.**, δg - closed set is a J* - closed set. To prove the other implications, in a $T_{1/2}$ - space,η^* - closed sets coincide with δ - closed sets (By **Proposition 3.13.**). Hence η^* - Cl(D) = δCl(D) $\subseteq$ M which implies J* - closed set is a δg - closed set. Hence J* - closed sets coincide with δg - closed sets. □

Example 8.2. Consider Y = {p,q,r},ζ = {ϕ, Y, {p}, {q,r}}.
Here J*C(Y,ζ) = δg C(Y,ζ) = P(Y). It is also a $T_{1/2}$-space.

Theorem 8.3. *If a space is a semi - regular space, then J* - closed sets and J - closed sets coincide.*

Proof. The proof follows from **Proposition 3.16.**. □

Remark 8.4. By **Theorem 7.3.** and **Corollary 7.6.** now J* - closed sets also coincide with g - closed, δg - closed, gδ - closed and δg‡ - closed sets.

Example 8.5. Consider Y={p,q,r},ζ={ϕ, Y, {p},{q,r}}. Here δC(Y,ζ) = η^*C(Y,ζ) = ζ^c and JC(Y, ζ) = J*C(Y,ζ) = gC(Y, ζ) = δgC(Y,ζ) =δg‡C(Y, ζ) = gδC(Y, ζ)= P(Y).

Remark 8.6. In general, a J* - closed set is independent with a closed set. But a space which is T_b - space and αT_b - space, every J* - closed set is a closed set. This can be seen in the following Propositions.

Proposition 8.7. If D is J* - closed, then it is closed when (Y,ζ) is a T_b - space.

Proof. By **Theorem 5.4.**, every J* - closed is gs - closed. In a T_b - space, every gs - closed is closed. Hence J* - closed is closed. □

Proposition 8.8. If D is J* - closed, then it is closed when (Y,ζ) is a αT$_b$ - space.

Proof. By **Theorem 5.4.**,every J* - closed is αg - closed. In a αT$_b$ - space, every αg - closed is closed. Hence J* - closed is closed. □

Theorem 8.9. *In an almost weakly Hausdorff space (Y,ζ),the g - closed sets of (Y,ζ_s)are δ - closed sets in (Y,ζ) and thus they are δg^* - closed sets, δg - closed sets, J* - closed sets respectively in (Y,ζ).*

Proof. By **Theorem 5.3.**,every δ - closed set is J* - closed. Hence to prove the given condition it is enough to prove every g- closed sets of(Y,ζ_s) are δ - closed in (Y,ζ). Let D $\subseteq$ Y be a g - closed subset of (Y,ζ_s).Consider x $\in$ δCl(D).If { x } is δ - open, then x $\in$ D. If not, thenY $\setminus$ {x} is δ - open,since Y is almost weakly Hausdorff.Let x $\notin$ D. By the given condition D is g - closed in (Y,ζ_s) then δCl(D) $\subseteq$ Y $\setminus$ {x},(i.e) x $\notin$ δ Cl(D). By contradicition x $\in$ D. ThenδCl(D) = D. Hence D is δ - closed hence by **Proposition 2.2.2.**[33],**Theorem 3.1.(i)** [8],**Theorem 5.3.** and D is δg* - closed sets, δg - closed sets, J* - closed sets respectively in (Y,ζ). □

Theorem 8.10. *Let D be a pre - open subset of a topological space (Y,ζ).Then the following conditions are equivalent:*
(a) D is δg - closed.
(b) D is J - closed.*

(c) D is g - closed.

Proof. $(a) \Rightarrow (b), (b) \Rightarrow (c)$ are true for any subset D by **Theorem 5.3.**, **Theorem 5.4.**. If D is pre - open in (Y,ζ), then by a result of [14],Cl(D) = δCl(D) and so $(c) \Rightarrow (a)$. □

Concerning partition spaces, the following characterization via J* - closed sets is obtained.

Corollary 8.11. Let D be a subset of the partition space (Y,ζ). Then the following conditions are equivalent:
(a) D is δg - closed.
(b) D is J* - closed.
(c) D is g - closed.

Proof. A topological space is a partition space if and only if every subset is pre - open.Thus the claim follows straight from **Theorem 8.10.**. □

Example 8.12. Let Y = {p,q,r},ζ = {ϕ, Y, {p}, {q,r}}. This is a partition space. It is seen that δgC(Y,ζ) = J*C(Y,ζ) = gC(Y,ζ) =P(Y).

Corollary 8.11. is modified by replacing the partition space by T_δ - space as follows.

Lemma 8.13. *If a topological space (Y,ζ) is a T_δ-space, then the following conditions are equivalent:*

(a) D is δg - closed.
(b) D is J - closed.*
(c) D is g - closed
(d) D is J - closed.
(e) D is gδ - closed.

Proof. $(a) \Rightarrow (b) and (b) \Rightarrow (c), (c) \Rightarrow (d)$ by **Theorem 5.3.**,**Theorem 5.4.**, **Theorem 4.4.** and **Theorem 4.8.** In T_δ-space, gδ - closed sets coincide with δ - closed sets and every δ - closed set is a δg - closed set by Theorem 3.1(i) of [8]. Hence $(d) \Rightarrow (a)$. □

Theorem 8.14. *If every subset of a T_b - space (resp.a αT_b - space) is J* - closed, then (Y,ζ) is a partition space.*

Proof. Let D be an open set. Since every subset is J* - closed,D is J* - closed. By **Proposition 8.7.**(resp. **Proposition 8.8.**) D is closed. Hence (Y,ζ) is a partition space. □

Theorem 8.15. *For a compact subset D of a R_1 - topological space (Y,ζ), every J* - closed set is a $\delta g^{\ddagger}$ - closed set. The converse is true if (Y,ζ) is semi - regular.*

Proof. Let D be J* - closed and D be a compact subset of R_1 - topological space (Y,ζ). Let D $\subseteq$ M, where M is δ - open. By **Theorem 3.3.**, every δ - open set is η^* - open .....(1).In R_1 - spaces the concepts of closure and δ - closure coincide for compact sets (By **Theorem 3.6.** of [15]). Hence for a compact subset D,δCl(D) =

Cl(D) .....(2). Generally δ - closed $\rightarrow$ η^* - closed $\rightarrow$ closed. Therefore Cl(D) $\subseteq$ η^*Cl(D) $\subseteq$ δCl(D).....(A). Hence (2) implies Cl(D) = η^*Cl(D)= δCl(D) .....(3). From (1) and (3) and since D is J* - closed, we get D is a δg‡ - closed set.
Conversely let (Y,ζ) be semi - regular and D be δg‡ - closed. Let D$\subseteq$ M, where M is open. By **Proposition 3.16.**, η^* - open sets coincide with δ - open sets and open sets in semi - regular spaces. From (A), η^*Cl(D) $\subseteq$ δCl(D) $\subseteq$ M, then D becomes J* - closed. □

Corollary 8.16. In Hausdroff spaces, also a finite set is J* - closed if and only if it is δg‡ - closed.

Proof. In a Hausdroff space, every finite set is closed. Hence the proof follows. □

9. Characterization theorems of J** - closed sets

Proposition 9.1. In a $T_{1/2}$ - space for a subset D the following are equivalent:
(i) D is J** - closed,
(ii) D is δg‡ - closed.

Proof. $(i) \Rightarrow (ii)$ Let D be J** - closed, D$\subseteq$ M where M is δ - open. In a $T_{1/2}$ - space, η^* - open sets coincide with δ - open sets (By **Proposition 3.13.**). Since D is J** - closed, η^*Cl(D)$\subseteq$ M,then by **Proposition 3.13.**,δCl(D)$\subseteq$ M $\Rightarrow$ D is δg‡ - closed.
$(ii) \Rightarrow (i)$ Let D be δg‡ - closed, D$\subseteq$ M where M is η^* - open. In a $T_{1/2}$ - space, η^* - open sets coincide with δ - open sets (By **Proposition 3.13.**). Since D is δg‡ - closed, δCl(D)$\subseteq$ M,then η^*Cl(D)$\subseteq$ δCl(D)$\subseteq$ M $\Rightarrow$ D is J** - closed. □

Proposition 9.2. If a space is a semi - regular space, then J** - closed sets,J* - closed sets, J - closed sets ,g - closed sets, δg - closed sets, gδ - closed sets and δg‡ - closed sets coincide.

Proof. In a semi - regular space, η^* - closed sets coincide with closed sets by **Proposition 3.16.** By **Theorem 8.3.** and **Remark 8.4.**, in a semi-regular space,J* - closed sets and J - closed sets coincide with g - closed sets, δg - closed sets, gδ - closed sets and δg‡ - closed sets. Hence the Proof. □

Example 9.3. Let Y = {p,q,r}, ζ = {Y, ϕ,{p},{q,r}}.Here Y is a semi-regular space,where JC(Y,ζ) =J*C(Y,ζ) = J**C(Y,ζ) = gC(Y,ζ) = δgC(Y,ζ) = δg‡C(Y,ζ) = gδC(Y,ζ) =P(Y).

Note 9.4. The converse of above **Proposition 9.2.** is not correct. It follows from the Example.

Example 9.5. Let Y = {p,q,r}, ζ = {Y, ϕ, {p}, {p,q}}. Here JC(Y,ζ) =P(Y) - {p} = J**C(Y,ζ) , J*C(Y,ζ) = gC(Y,ζ) = δg*C(Y, ζ) = δgC(Y, ζ) ={Y, ϕ,{r},{p,r},{q,r}} and δg‡C(Y, ζ) = gδC(Y, ζ) = P(Y) ,but Y is not a semi-regular space.

Remark 9.6. It is to be noted that in a semi - regular space gδ - closed sets conincide with J** - closed sets. But in a $T_{1/2}$ - space, g δ - closed set are different from J** - closed sets, whereas in a $T_{1/2}$ - space, gδ - closed sets coincide with J - closed sets in **Theorem 7.1.**

Example 9.7. Let Y = {p,q,r,s }, ζ = {Y, ϕ, {p}, {r}, {p,q}, {p,r}, {p,q,r}, {p,r,s}}. Here J**C(Y,ζ) = P(Y) - {{p}, {q}, {p,q}} $\neq$ gδC(Y,ζ) = P(Y) - {{p}, {r}, {p,q}, {q,r},{p,r}, {p,q,r}} but Y is a $T_{1/2}$ - space.

Theorem 9.8. *If a topological space (Y, ζ) is a T_δ - space, then the following conditions are equivalent:*

(a) D is δg - closed
(b) D is J - closed*
*(c) D is J** - closed*
(d) D is J - closed
(e) D is $g\delta$ - closed.

Proof. (a) $\Rightarrow$ **(b), (b)** $\Rightarrow$ **(c) , (c)** $\Rightarrow$ **(d)** and **(d)** $\Rightarrow$**(e)** by **Theorem 5.3., Theorem 6.4.**, **Theorem 6.5.**, **Theorem 4.8.** In T_δ-space, gδ-closed is δ-closed and δ-closed set is a δg-closed set by **Theorem 3.1.(i)** of [8]. Hence **(e)** $\Rightarrow$ **(a)**. □

Remark 9.9. In general J** - closed sets are independent with g - closed sets. But the following Theorem explains in a semi - regularspace, they are coincident.

Theorem 9.10. *Let D be a subset of a semi - regular space (Y,ζ) then D is J** - closed if and only if D is g - closed*

Proof. By **Proposition 3.16.**, η^* - closed sets coincide with δ - closed sets and closed sets. We reached the proof. □

Theorem 9.11. *In an almost weakly Hausdorff space (Y,ζ),the g - closed sets of (Y,ζ_s) are δ - closed sets in (Y,ζ) and thus δg^* - closed sets, δg - closed sets, J** - closed sets, J*- closed sets respectively in (Y,ζ).*

Proof. Let D $\subseteq$ Y be a g - closed subset of (Y,ζ_s). Let x $\in$ δCl(D). If {x} is δ - open, then x $\in$D. If not, then Y \{x} is δ - open, since Y is almost weakly Hausdorff. Assume that x $\notin$ D. Since D is g-closed in (Y,ζ_s) then δCl(D) $\subseteq$Y \{x},that is x $\notin$ δCl(D). By contradicition x $\in$ D.Then Cl(D) = D in (Y,ζ). Hence D is δ - closed. Since D is δ - closed and hence by **Proposition 2.2.2.**[33],**Theorem 3.1.(i)** [8],**Theorem 6.4.** and **Theorem 5.3.**, D is δg* - closed, δg - closed, J** - closed and J* - closed respectively in (Y,ζ). □

Remark 9.12. In general the concepts of J** - closed sets and $\delta g^\ddagger$-closed sets are not equivalent. But for a compact subset D of a space which is $T_{1/2}$ and R_1, the concepts of J** - closed sets and δ $g^\ddagger$ - closed sets coincide. This can be seen in the following Theorem.

Theorem 9.13. *For a compact subset D of a R_1 - topological space (Y,ζ),every J** - closed set is a $\delta g^\ddagger$ - closed set. The converse is true if (Y,ζ) is $T_{1/2}$ (resp.semi-regular).*

Proof. Let D be J^{**} - closed and D be a compact subset of R_1 - topological space (Y,ζ). Let D $\subseteq$ M, where M is δ - open. By **Theorem 3.3.**, every δ - open set is η^* - open .....(1). In R_1 - spaces the concepts of closure and δ-closure coincide for compact sets (By **Theorem 3.6.** of [10]). Hence for a compact subset D,δCl(D) = Cl(D) .....(2).

Generally δ - closed $\to$ η^* - closed $\to$ closed. Therefore Cl(D) $\subseteq$ η^*Cl(D) $\subseteq$ δCl(D) .....(A). Hence (2) implies Cl(D) = η^*Cl(D)= δCl(D) .....(3). From (1) and (3) and since D is J** - closed, we get D is a δg‡ - closed set.
Conversely let (Y,ζ) be $T_{1/2}$ and D be δ g ‡ - closed. Let D $\subseteq$ M, where M is η^* - open. By **Proposition 3.13.**(resp. **Proposition 3.16.**), η^* - open sets coincide with δ - open sets in $T_{1/2}$ - spaces (resp.semi-regular spaces). From (A), η^*Cl(D) $\subseteq$ δCl(D) $\subseteq$ M. Then D becomes J** - closed. □

Corollary 9.14. In Hausdroff spaces, a finite set is J** - closed if and only if it is δg‡ - closed.

10. Conclusion

1. In a $T_{1/2}$ - space, η^* - closed sets coincide with δ - closed sets. Moreover in this space, the family of J - closed sets is equivalent with that of gδ - closed sets and the family of J* - closed sets coincide with the family of δg - closed sets and the family of J**- closed sets coincide with the family of δg‡ - closed sets.
2. In a semi-regular space, δ - closed sets, η^* - closed sets and closed sets coincide. So δ - closure, η^* - closure, closure of any subset coincide. Therefore from definitions J** - closed sets, J* - closed sets,J - closed sets,g - closed sets, δg - closed sets, gδ - closed sets and δg‡ - closed sets coincide.
3. (*a*) In a $T_{1/2}$-space and a semi-regular space, J - closed sets are closed.
   (*b*) In T_b (resp. αT_b)- spaces,J* - closed sets are closed.
4. Regarding partition spaces we have obtained following results.
   (*a*) Every subset of a partition space is J - closed. The result fails for J* - closed sets and J** - closed sets.
   (*b*) If every subset of a space Y which is both $T_{1/2}$ and semi-regular is J - closed, then Y is also a partition space.
   (*c*) In a T_b - space (resp. αT_b - space) Y if every subset is J* - closed, then Y is a partition space.
   (*e*) Let D be a subset of the partition space (Y,ζ). Then the following conditions are equivalent:
   - D is δg - closed.
   - D is J* - closed.
   - D is g - closed.
5. (*a*) If a compact subset D of a R_1 - topological space (Y,ζ) is J - closed (resp. J* - closed,J** - closed), then D is a δg‡ - closed set.
   (*b*) The converse is true only when
   (*i*) Y is $T_{1/2}$ in the case of J - closed sets.
   (*ii*) Y is semi-regular in the case of J* - closed sets.
   (*iii*) Y is $T_{1/2}$ (resp.semi-regular) in the case of J** - closed sets.
6. In Hausdroff spaces, a finite set D is one of these three types namely J - closed set,J* - closed set or J** - closed set,then D is δg‡ - closed set.
7. In a T_δ - space, δg - closed, J** - closed, J* - closed, g - closed, J - closed, gδ - closed sets are equivalent.
8. In an almost weakly Hausdorff space (Y,ζ),the g-closed sets of (Y,ζ_s) are δ - closed sets in (Y,ζ) and thus δg* - closed sets, δg - closed sets,J** - closed sets, J* - closed sets respectively.

9. If D is a pre-open subset of a topological space (Y,ζ). Then the following conditions are equivalent:
   - D is δg - closed.
   - D is J* - closed.
   - D is g - closed.

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