Poincare Journal of Analysis & Applications
Vol. 8, No. 1(I) (2021), Special issue, 273-279
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DOI: 10.33786/pjaa.2021.v08i01(i).025

NANO gj -COMPACTNESS AND NANO gj-CONNECTEDNESS ON NANO TOPOLOGICAL SPACES

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Date of Receiving	:	28. 12. 2020
Date of Revision	:	06. 02. 2021
Date of Acceptance	:	22. 02. 2021

Abstract. In this paper, a new notion of nano gj-compactness and nano gj-connectedness are initiated and related theorems are proved. Moreover, we explain some properties with interesting examples.

1. Introduction

Topology is an important part of mathematics research area. In particular, nano topology is an escalating part of topology. Connectedness in topology was introduced by A.V.Arhangelskii, et.al.,[1]. The idea of compactness is an essential part of general topology. Many recent researchers contribute their work on connectedness and compactness in general topology.

Lellis Thivagar[2] introduced nano topology which was defined in terms of lower, upper approximations and boundary region using an equivalence relation on it. The idea of nano compactness and nano connectedness in nano topological space was given by S.Krishnaprakash, et.al.,[3]. Nano j-closed sets and nano gj-closed sets were launched by D.Sasikala and K.C.Radhamani[5].

In this article, nano gj-compactness and nano gj-connectedness on nano topological spaces are established.

$$\begin{aligned} \equiv 16 \times 16 &= 256 \\ &\equiv 25 \mid 6 \times 3 \\ &\equiv 25 \text{ Months } 18 \text{ days} \\ &\equiv 2\text{Y } 18 \text{ days.} \end{aligned}$$

2010 *Mathematics Subject Classification.* xxC15, xxA38.
Key words and phrases. Nano gj-open cover, nano gj-compact, nano gj-connected, nano gj-irresolute.
Communicated by. S. Jafari and P. Gananchandra
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2. Preliminaries

Definition 2.1[2] Let U be the universal set, a nonempty finite set of objects and R be an equivalence relation on U. Then (U, R) is said to be an approximation space.

Definition 2.2[2] Let $X \subseteq U$. The lower approximation of X with respect to the equivalence relation R is the set of all objects, which can be certainly classified as X with respect to R and is denoted by $L_R(X)$. That is $L_R(X) = \cup\{R(a) : R(a) \subseteq X, a \in U\}$, where $R(a)$ denotes the equivalence class determined by $a \in U$.

The upper approximation of X with respect to the equivalence relation R is the set of all objects, which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. That is $U_R(X) = \cup\{R(a) : R(a) \cap X \neq \emptyset, a \in U\}$, where $R(a)$ denotes the equivalence class determined by $a \in U$.

The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not as X with respect to R and is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.3[2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. The space $[U, \tau_R(X)]$ is called the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets.

Definition 2.4[2] If $[U, \tau_R(X)]$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $NInt(A)$.

That is $NInt(A)$ is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed set containing A and it is denoted by $NCl(A)$ which is the smallest nano closed set containing A.

Definition 2.5[3] A collection $\{A_i : i \in \Omega\}$ of nano-open sets in a nano topological space $[U, \tau_R(X)]$ is called a nano open cover of a subset B of U if $B \subset \{A_i : i \in \Omega\}$ holds.

Definition 2.6[3] A subset B of a nano topological space $[U, \tau_R(X)]$ is said to be nano compact relative to $[U, \tau_R(X)]$, if for every collections $\{A_i : i \in \Omega\}$ of nano open subsets of $[U, \tau_R(X)]$ such that $B \subset \{A_i : i \in \Omega\}$ there exists a finite subset Ω_0 of Ω such that $B \subset \{A_i : i \in \Omega_0\}$.

Definition 2.7[3] A subset B of a nano topological space $[U, \tau_R(X)]$ is said to be nano compact if B is nano compact as a subspace of $[U, \tau_R(X)]$.

Definition 2.8[3] A nano topological space $[U, \tau_R(X)]$ is said to be nano connected if $[U, \tau_R(X)]$ cannot be expressed as a disjoint union of two non-empty nano open sets. A subset of $[U, \tau_R(X)]$ is nano connected as a subspace. A subset is said to be nano disconnected if and only if it is not nano connected.

Definition 2.9[5] A subset of a nano topological space $[U, \tau_R(X)]$ is called a nano j-open set if $A \subseteq NInt[NPCl(A)]$. The complement of nano j-open set is called a nano j-closed

(briefly Nj-closed) set. If A is Nj-closed, then $NCl[NPInt(A)] \subseteq A$.
Definition 2.10[5] A subset of a nano topological space $[U, \tau_R(X)]$ is called a nano generalized j-closed (briefly Ngj-closed) set if $NJCl(A) \subseteq V$ where $A \subseteq V$ and V is nano open in U.

3. Nano gj-Compactness

Definition 3.1 A collection $\{A_i : i \in \Omega\}$ of nano gj-open sets in a nano topological space $[U, \tau_R(X)]$ is called a nano gj-open cover of a subset B of U if $B \subset \cup\{A_i : i \in \Omega\}$.
Example 3.2 Let $U = \{p, q, r, s\}$ with $U\backslash R = \{\{q\}, \{r\}, \{p, s\}\}$. Let $X = \{p, r\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{r\}, \{p, s\}, \{p, r, s\}\}$. The nano gj-open sets are $U, \emptyset, \{p\}, \{r\}, \{s\}, \{p, r\}, \{p, s\}, \{r, s\}, \{p, r, s\}$. The collection $\{\{p\}, \{s\}, \{p, r, s\}\}$ is a nano gj-open cover of the subset $\{p\}$ since $\{p\} \subset \cup\{\{p\}, \{s\}, \{p, r, s\}\}$.
Definition 3.3 A nano topological space $[U, \tau_R(X)]$ is countably nano gj-compact if every countable nano gj-open cover of U has a finite sub-cover.
Example 3.4 Let $U = \{p, q, r, s, t\}$ with $U\backslash R = \{\{p, q\}, \{r, t\}, \{s\}\}$. Let $X = \{p, s\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{s\}, \{p, q\}, \{p, q, s\}\}$. The nano gj-open sets are $U, \emptyset, \{p\, \{q\}, \{s\}, \{t\}, \{p, q\}, \{p, r\}, \{p, s\}, \{p, t\}, \{q, r\}, \{q, s\}, \{q, t\}, \{r, s\}, \{s, t\}, \{p, q, s\}$, $\{p, r, s\}, \{p, q, r\}, \{p, q, t\}, \{p, s, t\}, \{q, r, t\}, \{q, s, t\}, \{p, q, r, s\}, \{p, q, s, t\}$. Here $[U, \tau_R(X)]$ is countably nano gj-compact, since every countable nano gj-open cover of U has a finite sub-cover.
Definition 3.5 A subset B of a nano topological space $[U, \tau_R(X)]$ is said to be nano gj-compact relative to $[U, \tau_R(X)]$, if every collection $\{A_i : i \in \Omega\}$ of nano gj-open subsets of U such that $B \subset \cup\{A_i : i \in \Omega\}$, there exists a finite subset Ω_o of Ω such that $B \subset \cup\{A_i : i \in \Omega_0\}$.
Definition 3.6 A subset B of a nano topological space $[U, \tau_R(X)]$ is said to be nano gj-compact if B is nano gj-compact as a subspace of U.
Theorem 3.7 Every nano gj-closed subset of a nano gj-compact space $[U, \tau_R(X)]$ is nano gj-compact relative to $[U, \tau_R(X)]$.
Proof. Let A be a nano gj-closed subset of the nano gj-compact space $[U, \tau_R(X)]$. Then A^c is nano gj-open in U. Let $M = \{G_\alpha : \alpha \in \Omega\}$ be a nano-open cover of A by nano gj-open sets in U. Then $M^\star = M \cup A^c$ is a nano gj-open cover of U. Since U is nano gj-compact, $M^\star$ is reducible to a finite sub cover of U, such that $U = G_{\alpha 1} \cup G_{\alpha 2} \cup ... G_{\alpha m} \cup A^c$,$G_{\alpha k} \in M$. But A and A^c are disjoint. Hence $A \subset G_{\alpha 1} \cup G_{\alpha 2} \cup ... G_{\alpha m}$, $G_{\alpha k} \in M$, which implies that any nano gj-open cover M of A contains a finite subcover. Therefore A is nano gj-compact relative to U. Thus every nano gj-closed subset of a nano gj-compact space $[U, \tau_R(X)]$ is nano gj-compact relative to $[U, \tau_R(X)]$.
Definition 3.8 A function $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ is said to be nano gj-continuous if $f^{-1}(B)$ is nano gj-open in U for every nano open set B of V.
Definition 3.9 A function $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ is said to be nano gj-irresolute if $f^{-1}(B)$ is nano gj-open in U for every nano gj-open set B of V.
Theorem 3.10 A nano gj-continuous image of nano gj-compact space is nano compact.
Proof. Let $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ be a nano gj-continuous map from a nano gj-compact space $[U, \tau_R(X)]$ onto a nano topological space $[V, \tau_{R'}(Y)]$. Let $\{A_i : i \in \Omega\}$ be a nano open cover of $[V, \tau_{R'}(Y)]$. Then $\{f^{-1}(A_i) : i \in \Omega\}$ is a nano gj-open cover of U. Since U is nano gj-compact, it has a finite subcover $\{f^{-1}(A_1), f^{-1}(A_2), ..., f^{-1}(A_n)\}$. Since f is onto and $\{A_1, A_2, ..., A_n\}$ is a cover of $[V, \tau_{R'}(Y)]$, which is finite. Hence

$[V, \tau_{R'}(Y)]$ is nano compact.

Theorem 3.11 If a map $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ is nano gj-irresolute and a subset B of $[U, \tau_R(X)]$ is nano gj-compact relative to $[U, \tau_R(X)]$, then the image $f(B)$ is nano gj-compact relative to $[V, \tau_{R'}(Y)]$.

Proof. Let $\{A_i : i \in \Omega\}$ be any collection of nano gj-open subsets of $[V, \tau_{R'}(Y)]$ such that $f(B) \subset \cup\{A_i : i \in \Omega\}$. Then $B \subset \cup\{f^{-1}(A_i) : i \in \Omega\}$ holds. Since by hypothesis B is nano gj-compact relative to $[U, \tau_R(X)]$, there exists a finite nano subsets Ω_0 of Ω such that $B \subset \cup\{f^{-1}(A_i) : i \in \Omega_0\}$. Therefore we have $f(B) \subset \cup\{A_i : i \in \Omega_0\}$, which shows that f(B) is nano gj-compact relative to $[V, \tau_{R'}(Y)]$.

Theorem 3.12 A space $[U, \tau_R(X)]$ is nano gj-compact if and only if each family of nano gj-closed subsets of $[U, \tau_R(X)]$ with the finite intersection property has a non-empty intersection.

Proof. Given a collection S of subsets of $[U, \tau_R(X)]$, let $C = \{U - A, A \in S\}$ be the collection of their complements. Then the following statements hold.

a) S is a collection of nano gj-open sets if and only if C is a collection of nano gj-closed sets.

b) The collection S covers $[U, \tau_R(X)]$ if and only if the intersection $\cap_{c \in C} C$ of all the corresponding elements of C is non-empty.

c) The finite sub collection $\{A_1, A_2, ..., A_n\}$ of S covers U if and only if the intersection of the corresponding elements $C_i = U - A_i$ of C is empty. The statement (a) is trivial, while (b) and (c) follow from De Morgan's Law. $U - (\cup_{i \in J} A_i) = \cap_{i \in J}(U - A_i)$. The proof of the theorem now follows in two steps, taking contra positive of the theorem and then the complement. Since U is nano gj-compact, for any given collection S of nano gj-open subsets of U, if S covers U, then some finite sub collection of S covers U. This statement is equivalent to its contra positive:

Given any collection S of nano gj-open sets, if no finite sub collection of S covers U, then S does not cover U. Let C be the collection of nano gj-closed sets, then the statement is equivalent to:

Given any collection C of nano gj-closed sets, if every finite intersection of elements of C is non-empty, then the intersection of all the elements of C is non-empty. Hence the theorem is proved.

Definition 3.13 A nano topological space $[U, \tau_R(X)]$ is said to be nano gj-Lindelof space if every nano open cover of $[U, \tau_R(X)]$ by nano gj-open sets contains a countable sub-cover.

Theorem 3.14 Let $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ be a nano gj-continuous surjection and $[U, \tau_R(X)]$ be nano gj-Lindelof, then $[V, \tau_{R'}(Y)]$ is nano Lindelof space.

Proof. Let $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ be a nano gj-continuous surjection and $[U, \tau_R(X)]$ be nano gj-Lindelof. Let $\{V_i\}$ be a nano open cover for $[V, \tau_{R'}(Y)]$. Then $\{f^{-1}(V_i)\}$ is a nano cover of $[U, \tau_R(X)]$ by nano gj-open sets. Since $[U, \tau_R(X)]$ is nano gj-Lindelof, $\{f^{-1}(V_i)\}$ contains a countable sub-cover, namely $\{f^{-1}(V_{in})\}$. Then V_{in} is a countable sub-cover of $[V, \tau_{R'}(Y)]$. Thus $[V, \tau_{R'}(Y)]$ is nano Lindelof space.

Theorem 3.15 Let $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ be a nano gj-irresolute surjection and $[U, \tau_R(X)]$ be nano gj-Lindelof, then $[V, \tau_{R'}(Y)]$ is nano gj-Lindelof space.

Proof. Let $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ be a nano gj-irresolute surjection and $[U, \tau_R(X)]$ be nano gj-Lindelof. Let $\{V_i\}$ be a nano open cover for $[V, \tau_{R'}(Y)]$. Then $\{f^{-1}(V_i)\}$ is a nano cover of $[U, \tau_R(X)]$ by nano gj-open sets. Since $[U, \tau_R(X)]$ is nano gj-Lindelof,

$\{f^{-1}(V_i)\}$ contains a countable sub-cover, namely $\{f^{-1}(V_{in})\}$. Then $\{V_{in}\}$ is a countable sub-cover of $[V, \tau_{R'}(Y)]$. Thus $[V, \tau_{R'}(Y)]$ is nano gj-Lindelof space.

4. Nano gj-Connectedness

Definition 4.1 A nano topological space $[U, \tau_R(X)]$ is said to be nano gj-connected if $[U, \tau_R(X)]$ cannot be expressed as a union of two disjoint non-empty nano gj-open sets. A subset of $[U, \tau_R(X)]$ is nano gj-connected if it is nano gj-connected as a subspace. A subset is said to be nano gj-disconnected if and only if it is not nano gj-connected.

Example 4.2 Let $U = \{p, q, r, s\}$ with the equivalence relation $U\backslash R = \{\{p, s\}, \{q\}, \{r\}\}$ and $X = \{p, r\}$. Then the nano topology is $\tau_R(X) = \{U, \emptyset, \{r\}, \{p, s\}, \{p, r, s\}\}$. Here the nano gj-open sets are $U, \emptyset, \{p\}, \{r\}, \{s\}, \{p, r\}, \{p, s\}, \{r, s\}, \{p, r, s\}$. Since U cannot be written as the union of two disjoint nano gj-open sets, $[U, \tau_R(X)]$ is nano gj-connected.

Theorem 4.3 For a nano topological space $[U, \tau_R(X)]$, the following are equivalent.

i) $[U, \tau_R(X)]$ is nano gj-connected.

ii) U and $\emptyset$ are the only subsets of $[U, \tau_R(X)]$ which are both nano gj-open and nano gj-closed.

iii) Each nano gj-continuous map of $[U, \tau_R(X)]$ into a discrete space $[V, \tau_{R'}(Y)]$ with at least two points is a constant map.

Proof. (i) $\implies$ (ii)

Let O be any nano gj-open and nano gj-closed subset of $[U, \tau_R(X)]$. Then $U - O$ is both nano gj-open and nano gj-closed. Thus $U = O \cup (U - O)$, which is disjoint union of two non-empty nano gj-open sets, which contradicts the fact (i). Hence, either $O = \emptyset$ or $O = U$.

$(ii) \implies (i)$

Suppose that $U = A \cup B$ where A and B are disjoint non-empty nano gj-open subsets of $[U, \tau_R(X)]$. Since $A = U - B$, then A is both nano gj-open and nano gj-closed. By assumption, $A = \emptyset$ or $A = U$, which is a contradiction. Therefore U is nano gj-connected.

$(ii) \implies (iii)$

Let $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ be a nano gj-continuous map, where V is a discrete space with at least two points. Then $f^{-1}(V)$ is nano gj-closed and nano gj-open for each $v \in V$. That is $[U, \tau_R(X)]$ is covered by nano gj-open and nano gj-closed covering $\{f^{-1}(v) : v \in V\}$. By assumption $f^{-1}(v) = \emptyset$ or U , for each $v \in V$. If $f^{-1}(v) = \emptyset$ for all $v \in V$, then f fails to be a map. Then there exists only one point $v \in V$ such that $f^{-1}(v) \neq \emptyset$ and hence $f^{-1}(v) = U$. This shows that f is a constant map.

$(iii) \implies (ii)$

Let O be both nano gj-open and nano gj-closed in $[U, \tau_R(X)]$. Suppose $O \neq \emptyset$. Let $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ be a nano gj-continuous map defined by $f(O) = \{u\}$ and $f(O^c) = \{w\}$ for some distinct points u and w in V. By assumption, f is a constant map. Therefore we have $O = U$.

Theorem 4.4 Every nano gj-connected space is nano connected.

Proof. Let $[U, \tau_R(X)]$ be nano gj-connected. Suppose U is not nano connected. Then there exists a proper non-empty subset A of U which is both nano open and nano closed in U. Since every nano closed set is nano gj-closed, A is a proper non-empty subset of U which is both nano gj-open and nano gj-closed in U. By theorem 4.3, U is not nano gj-connected, which is a contradiction to our assumption. Hence every nano gj-connected space is nano connected.

Remark 4.5 The converse of the above theorem need not be true in general, which is shown in the following example.

Example 4.6 Let $U = \{p, q, r, s, t\}$ with $U \backslash R = \{\{p, q\}, \{r, t\}, \{s\}\}$ and let $X = \{p, s\}$. Then the nano topology $\tau_R(X) = \{U, \emptyset, \{s\}, \{p, q\}, \{p, q, s\}\}$. The nano open sets are U, $\emptyset, \{s\}, \{p, q\}, \{p, q, s\}$. The nano gj-open sets are $U, \emptyset, \{p\}, \{q\}, \{r\}, \{s\}, \{t\}, \{p, q\}\{p, r\}$, $\{p, s\}, \{p, t\}, \{q, r\}, \{q, s\}, \{q, t\}, \{r, s\}, \{s, t\}, \{p, q, r\}, \{p, q, t\}, \{p, s, t\}, \{p, q, s\}, \{p, r, s\}$, $\{q, s, t\}, \{q, r, s\, \{p, q, r, s\}, \{p, q, s, t\}$. Here U is nano connected, but it is not nano gj-connected since it can be written as the union of two disjoint non-empty nano gj-open sets $\{r\}$ and $\{p, q, s, t\}$. Thus, a nano connected space need not be nano gj-connected.

Theorem 4.7 If $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ is a nano gj-continuous surjection and U is nano gj-connected, then V is nano connected.

Proof. Suppose that V is not nano connected. Let $V = X \cup Y$ where X and Y are disjoint non-empty nano open sets in V. Since f is nano gj-continuous and onto, $U = f^{-1}(X) \cup f^{-1}(Y)$ where $f^{-1}(X)$ and $f^{-1}(Y)$ are disjoint non-empty nano gj-open sets in U. This contradicts the fact that U is nano gj-connected. Hence V is nano connected.

Theorem 4.8 If $f : [U, \tau_R(X)] \to [V, \tau_{R'}(Y)]$ is a nano gj-irresolute surjection and U is nano gj-connected, then V is nano gj-connected.

Proof. Suppose that V is not nano gj-connected. Let $V = X \cup Y$ where X and Y are disjoint non-empty nano gj-open sets in V. Since f is nano gj-irresolute and onto, $U = f^{-1}(X) \cup f^{-1}(Y)$ where $f^{-1}(X)$ and $f^{-1}(Y)$ are disjoint non-empty nano gj-open sets in U. This contradicts the fact that U is nano gj-connected. Hence V is nano gj-connected.

Theorem 4.9 In a nano topological space $[U, \tau_R(X)]$ with at least two points, if $NJO(U, \tau_R(X)) = NJCl(U, \tau_R(X))$, then U is not nano gj-connected.

Proof. Assume that $[U, \tau_R(X)]$ be a nano topological space with at least two points. Let $JO(U, \tau_R(X)) = JCl(U, \tau_R(X))$. Since every nano j-closed set is nano gj-closed, there exists some non-empty proper subset of U which is both nano gj-open and nano gj-closed in U. By theorem 4.3, U is not nano gj-conneted.

Definition 4.10 A nano topological space $[U, \tau_R(X)]$ is said to be nano T_{gj}-space if every nano gj-closed subset of U is nano closed subset of U.

Theorem 4.11 Suppose that U is a nano T_{gj}-space, then U is nano connected if and only if it is nano gj-connected.

Proof. Suppose that U is nano connected. Then U cannot be expressed as a disjoint union of two non-empty proper subsets of U. Suppose U is not a nano gj-connected space. Let X and Y be any two nano gj-open subsets of U, such that $U = X \cup Y$, where $X \cap Y = \emptyset$ and $X \subset U$, $Y \subset V$. Since U is nano T_{gj}-space and X,Y are nano gj-open, X and Y are nano open subsets of U, which contradicts that U is nano connected. Therefore U is nano gj-connected. Conversely, every nano open set is nano gj-open. Therefore every nano gj-connected space is nano connected.

Theorem 4.12 If the nano gj-open sets S and T form a nano separation of U and if V is nano gj-connected subspace of U, then V lies entirely within S or T.

Proof. Since S and T are both nano gj-open in U, the sets $S \cap V$ and $T \cap V$ are nano gj-open in V. These two sets are disjoint and their union is V. If both the sets are non-empty, they will constitute a separation of V. Therefore, one of them is empty. Hence V must lie entirely within S or in T.

Theorem 4.13 Let X be a nano gj-connected subspace of U. If $X \subset Y \subset NgjCl(X)$ then Y is also nano gj-connected.

Proof. Let X be nano gj-connected and let $X \subset Y \subset NgjCl(X)$. Suppose that $Y = S \cup T$ is a separation of Y by nano gj-open sets. Then by theorem 4.12, X must lie entirely in S or in T. Suppose that $X \subset S$, then $NgjCl(X) \subseteq NgjCl(S)$. Since $NgjCl(S)$ and T are disjoint, Y cannot intersect T. This contradicts the fact that T is non-empty subset of Y. Therefore $T = \emptyset$ which implies Y is nano gj-connected.

5. Conclusion

This article introduces nano gj-compactness and nano gj-connectedness and proves the theorems that bear relation between nano gj-compactness and nano compactness. To the newly introduced concepts relevant examples were also given and this work may leads to nano gj-compactifications.

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